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The response of exchange rates to permanent and transitory shocks under floating exchange rates

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Using the joint behavior of inflation and real exchange rates, we develop an empirical model to uncover the sources of the fluctuations in the real dollar exchange rates of four major industrial countries under the current float. This model allows us to construct two time series for each country pair, one representing the permanent component of each real exchange rate, and the other the purely transitory component. Over the period as a whole, transitory shocks played a relatively small but statistically significant role. Real dollar exchange rates therefore did not simply evolve in response to permanent shocks. Instead, there are instances in which temporary shocks made a substantial contribution. We conclude that the random walk model, though an approximate statistical description of real-exchange-rate behavior, is a poor guide to model structure. (JEL F31).

Traditional explanations of exchange-rate behavior have not fared well over the past two decades. Simple time series models have generally outperformed theoretically based models of exchange rates in forecasting nominal exchange rates during the floating-rate period. Movements in real and nominal exchange rates have been highly correlated. Perhaps most importantly, in all but a few instances, researchers have been unable to reject the hypothesis that real exchange rates have followed random walks during this period. As a consequence, purchasing power parity is now regarded by many, if not most, researchers as of virtually no use empirically. To explain exchange-rate movements under the

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float, there has been widespread resort to models in which real shocks play the dominant role.[1]

Our purpose in this paper is to investigate these issues. To do so, we study the time-series properties of the real exchange rates of Germany, Italy, Japan and the UK relative to the US dollar and of the inflation rates of all five countries over the period 1974 to 1989. We develop an empirical model that uses the joint behavior of inflation and real exchange rates to back out the role played by the different shocks affecting real exchange rates. This model allows us to uncover the sources of the fluctuations in real exchange rates with greater precision than is possible from studying real exchange rates alone.

Our model uncovers the sources of exchange rate fluctuations by classifying different structural shocks according to their long-run impacts on the real exchange rate. It allows us to construct two time series for each country pair, one representing the 'permanent' (or highly persistent) component of each real exchange rate, and the other the purely transitory component. These estimates speak directly to the question of whether real exchange rates do in fact follow random walks. They also permit us to make inferences with regard to the possible sources of exchange-rate movements over these years.

Several sets of findings emerge from this analysis. Most importantly, the data indicate that over the whole floating-rate period transitory shocks had a relatively small but statistically significant influence on real exchange rates. Real dollar exchange rates have not simply evolved in response to permanent shocks. Instead, there are instances – depending upon the currency and the time period – where temporary shocks appear to have made a substantial contribution to their evolution. Thus, it appears that the random walk model is a poor guide to model structure even though it approximately describes the time-series behavior of real exchange rates over the whole floating-rate period.

The variations in the relative contributions of the permanent and temporary components give some important clues about the underlying sources of the shocks to real exchange rates. Differences in temporary components parallel observed differences in the conduct of monetary policy, both over time and across countries. We also find that the contribution of the permanent component is heavily concentrated in one important and lengthy episode, the substantial dollar appreciation, and then roughly offsetting the depreciation of the 1980s.[2] Because these two swings are characteristic of all four real exchange rates, they appear to be US related. Clearly, they are not due to the monetary shocks of overshooting models. Nor do they seem to us to be capable of description in terms of the productivity shocks of equilibrium models.

I. Exchange-rate theory and empirical regularities

We define the real exchange rate in terms of the deviation from purchasing power parity (PPP):

$$q_t = e_t - (p_t^* - p_t), \quad \langle 1 \rangle$$

where q is the logarithm of the real exchange rate, e is the logarithm of the nominal rate (the price in foreign currency of a unit of domestic currency), p^* and p are logarithms of the foreign and domestic price levels, respectively, and t is a time subscript. If PPP holds continuously in absolute form, q will be constant and identically zero. If PPP only holds in relative form, q still will be constant but non-zero. Given that data on actual price levels and on cross-country price indices usually are unobtainable, most researchers have investigated this latter version of the hypothesis.[3]

Conceptually it is useful to think about movements in real exchange rates as made up of fluctuations about long-run equilibrium values and changes in those equilibrium values themselves. We can define the latter using as a direct analogue of ⟨1⟩:

$$\langle 2\rangle \qquad \bar{q}_t = \bar{e}_t - (\bar{p}_t^* - \bar{p}_t),$$

where a bar over a variable represents a long-run equilibrium value. By subtracting ⟨2⟩ from ⟨1⟩ and rearranging terms we can express the deviation of (actual) q_t from its long-run equilibrium value, $\bar{q}_t$, as:

$$\langle 3\rangle \qquad q_t - \bar{q}_t = (e_t - \bar{e}_t) - [(p_t^* - \bar{p}_t^*) - (p_t - \bar{p}_t)],$$

or more simply,

$$\langle 4\rangle \qquad q_t = \bar{q}_t + (q_t - \bar{q}_t),$$

where the term in parentheses is defined as in equation ⟨3⟩.

If (relative) PPP holds in this long-run context, $\bar{q}_t$ will be a constant, call it $\bar{q}_0$ and q_t will ultimately converge to this value. This in turn implies convergence of e_t, p_t, and p_t^* to their equilibrium values, the algebraic sum of which will of course equal $\bar{q}_0$ too. Over the short-run, however, q_t need not, and empirically generally will not, equal $\bar{q}_0$. Divergences between the two will exist so long as e_t, p_t, and p_t^* diverge from their long-run equilibria. As a result, tests of long-run PPP have increasingly focused on the error process followed by q_t, and in particular whether q_t contains a unit root, or does in fact show convergence to some stable value.

With the exception of 'equilibrium' models like those developed by Lucas (1982) and Stockman (1980), most exchange-rate models allow for such short-run deviations about the long-run equilibrium. In the literature, these deviations typically are viewed as monetary in origin, given rational expectations, the result of unanticipated changes in excess supplies of money. With extremely rapid adjustment of the nominal exchange rate and a posited sluggish adjustment of price levels, these monetary shocks lead to short-run fluctuations in the actual real exchange, as a result both of the differing speeds of adjustment and the related short-run overshooting of the nominal exchange rate (Dornbusch, 1976; Mussa, 1982). In the long run, however, the price level in the country experiencing the shock and the nominal exchange rate adjust by the same proportion.

A variety of theoretical models allow for the possibility of permanent shifts in $\bar{q}_t$, typically as a result of changes in real variables. Differences in rates of productivity growth among countries, productivity shocks, changes in relative prices, and changes in the terms of trade have all been cited in the literature as influences that will permanently affect the real exchange rate.[4]

While this identification of permanent shocks with real variables and transitory with nominal variables may be a useful first approximation, we do not regard it as an infallible guide to the source of real world shocks. In principle, real variables could in some instances have purely temporary effects. Consider, for example, a transitory decline in real income that lowered durable consumption and spilled over into the trade account temporarily affecting the real exchange rate. By the same token, shifts in monetary (and inflation) regime, such as those that took place in the USA in the post-Second World War period, could have very long lasting effects.

Roll (1979), Darby (1983) and Adler and Lehman (1983), in widely-cited earlier studies using time-series models, concluded that real exchange rates under the float, rather than reverting to constant values, were approximately random walks. Subsequent research applying univariate unit-root tests to q and related tests for cointegration of e and $(p-p^*)$ has largely supported those claims. Most researchers have been unable to reject the hypotheses that the processes governing real exchange rates under the float contain unit roots, or correspondingly that nominal exchange rates and relative price levels are not cointegrated.[5] As Darby pointed out, relative PPP in this case can still be a useful long-run concept in the sense that rates of change of nominal exchange rates will ultimately converge to inflation differentials. Of course, the usefulness of this concept will depend upon the relative incidence and magnitude of permanent shocks.

The substantial and seemingly erratic variations in nominal rates, particularly nominal US dollar rates under the float, greatly increased skepticism about the empirical content of PPP and relatedly the importance of (transitory) monetary shocks in explaining movements in both nominal and real exchange rates. The evidence of Meese and Rogoff (1983) that a simple random walk model of the nominal rate performed as well or better than theoretically based models in predicting nominal exchange rates during this period has added to this sentiment.

Especially troubling have been the wide swings in the dollar in the 1980s. Table 1 provides an overview of these movements. Shown there are cumulative percentage changes of the International Monetary Fund's indices of real and nominal effective dollar exchange rates for four sub-periods, along with the differences between the two, the differential percentage changes of the foreign and the US price indices (measured here by unit labor costs). What stands out are the substantial variations in both nominal and real dollar exchange rates during these years – particularly in the period up to mid-1988 – and the close relationship that changes in the one have borne to changes in the other. Within each of the sub-periods, therefore, changes in nominal dollar exchange rates and inflation differentials appear to have been virtually independent.

TABLE 1. Average changes in real and nominal effective dollar exchange rates and relative price levels: 1980–1990

Period	Exchange Rates		Relative Price Levels
	Real	Nominal	
1980:Q2–1985:Q1	44.5%	45.8%	−1.3%
1985:Q1–1988:Q2	−54.8%	−51.5%	3.4%
1988:Q1–1989:Q3	9.2%	7.3%	−1.9%
1989:Q3–1990:Q4	−19.3%	−16.1%	3.2%

Source: IMF, *International Financial Statistics*.
Note: Figures are cumulative percentage changes computed as the differences in the logarithm of the ending and beginning quarters of each sub-period multiplied by 100. Price levels are measured by unit labor costs.

Despite considerable effort on the part of many researchers there is, however, no widely accepted explanation for the dollar's behavior. Empirical studies aimed at finding one have either been completely unsuccessful, if not initially then subsequently, as later swings in the dollar appeared at variance with the previously maintained hypothesis, or so specifically dollar-focused that tests of their general validity were impossible.[6]

The poor performance of PPP observed in the 1980s is not, however, mirrored in other data. A number of recent studies applying cointegration tests or their univariate counterparts to exchange-rate data from other, mostly much longer, periods have achieved decidedly more positive results. Very long-term time series generally reject the hypotheses that relative price levels and nominal exchange rates are not cointegrated and correspondingly that the real exchange rate is not non-stationary (in some instances, most notably the yen/dollar and yen/sterling rates, not non-trend-stationary).[7] With shorter-term time series for periods and countries other than the industrialized countries under the float, researchers sometimes also are able to reject these hypotheses.[8]

One reason for the disparity in results achieved with the different bodies of data, as a number of researchers have suggested, is that the time spanned by the data, rather than the number of observations per se, may be important.[9] Given what appear to be quite slow speeds of adjustment – half lives of the order of two to three years – a decade and a half or so of experience under the float simply may be too short a time span, regardless of the frequency of the data, to discriminate between slow reversion to equilibrium and unit-root behavior.[10]

An alternative, though not mutually exclusive, explanation has to do with relative variance of the shocks that affect exchange rates. Both may indeed always be of some importance, with the degree of importance varying over time. Over very long periods (as also during extreme inflation episodes) temporary nominal shocks may very well predominate, while over shorter periods real shocks may be relatively more important. Unit-root tests, like other hypothesis tests, are of the either–or variety. Depending on the relative variance of the two types of shocks they may easily lead to different conclusions with different bodies of data.

Our focus, therefore, is on separating the contributions of temporary and permanent shocks. In doing so we address the related questions of PPP performance under the float and correspondingly whether a new model to explain exchange-rate behavior during this period is required.

II. Statistical methods

Our statistical methodology for examining the permanent and temporary components of real exchange rates builds upon the methods developed in Blanchard and Quah (1989), Shapiro and Watson (1988) and Evans (1991). We examine an empirical model for the first differences of the real exchange rate Δq_t, the domestic inflation rate Δp_t, and the foreign inflation rate Δp_t^*. This model uncovers the sources of exchange rate and inflation fluctuations by classifying different structural shocks according to their long-run impact on the real exchange rate. We use the model estimates to calculate the relative contribution of temporary shocks to the variance of real exchange rates. The model estimates are also used to trace the hypothetical path the real exchange rate would have followed if all shocks to exchange rates were temporary.

We might begin our search for evidence of temporary components in real exchange rates by looking for a moving average structure in their first difference. However, since previous research has established that the level of the real exchange rate is well approximated by a random walk, there is little prospect of being able to find a statistically significant moving average structure. This suggests that either temporary shocks are absent, or that their presence cannot be detected by examining real exchange rates alone because those data contain too much 'noise' from the permanent shocks to detect the effects of temporary shocks, the 'signal'.

The model we develop uses the joint behavior of inflation and real exchange rates to identify the sources of exchange rate fluctuations. If temporary shocks, like monetary shocks, affect real exchange rates, in all likelihood they should also influence inflation. We should therefore expect to find that month-to-month and quarter-to-quarter (*i.e.* high frequency) changes in the two variables will be correlated. Thus, insofar as fluctuations in inflation reflect the impact of temporary shocks that also affect real exchange rates, the joint behavior of inflation and exchange rates should provide more information on the role of temporary shocks than exchange rates alone. In other words, we exploit the high frequency correlation between the two variables to increase the signal to noise ratio, and thus better identify the contribution of the temporary shocks to real exchange rates.

II.A Empirical model

We assume that the dynamics of $\Delta X_t \equiv [\Delta q_t, \Delta p_t, \Delta p_t^*]$ follow

$$\langle 5 \rangle \qquad \Delta X_{t+1} = A_t(L)U_{t+1}, \qquad E_t U_{t+1} U'_{t+1} = \Sigma_t,$$

where $A_t(L)$ is a matrix polynomial in the lag operator. Our empirical model allows for changes in the dynamics of exchange and inflation rates through changing coefficients in $A_t(L)$. These coefficients can be viewed as reduced-form parameters that aggregate individual decisions. As such, they may well vary over time for the reasons made popular by Lucas.

U_t is a vector of structural shocks affecting exchange rates and inflation rates. The vector may include domestic and foreign productivity shocks, price shocks, nominal and real demand shocks. Theoretically speaking, some of these structural disturbances, like nominal demand shocks, should only have temporary effects on q_t. Others, such as taste and productivity shocks, may have permanent effects. Since we are primarily interested in the relative contribution of temporary versus permanent shocks to the real exchange rate, we shall not attempt to identify the exact source of these shocks. Instead, we assume that U_t comprises a vector of four shocks $[u_{1,t}, u_{1,t}^*, u_{2,t}, u_{2,t}^*]'$ where $u_{1,t}$ and $u_{1,t}^*$ are temporary domestic and foreign shocks, and $u_{2,t}$ and $u_{2,t}^*$ are permanent domestic and foreign shocks.

We assume that temporary and permanent shocks are uncorrelated. Since monetary shocks are examples of temporary shocks and policy changes are sometimes coordinated between central banks, we allow the temporary shocks to be correlated across countries. Permanent shocks are assumed to be contemporaneously uncorrelated across countries because in the monthly data that we analyze there seems little *a priori* reason to suspect that things like taste and productivity shocks should be. We also normalize all the shocks to have unit variances, so their covariance matrix is given by

$$\Sigma_t \equiv \begin{bmatrix} 1 & \rho_t & 0 & 0 \\ \rho_t & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

We are interested in the relative contribution of the temporary shocks, $u_{1,t}$ and $u^*_{1,t}$, to the real exchange rate, q_t. The contemporaneous effects of U_t on X_t are determined by the impact matrix:

$$\langle 6 \rangle \qquad A_t(0) = \begin{bmatrix} a_{1,t} & a_{2,t} & a_{3,t} & a_{4,t} \\ a_{5,t} & 0 & a_{6,t} & 0 \\ 0 & a_{7,t} & 0 & a_{8,t} \end{bmatrix}.$$

All four structural shocks affect the real exchange rate contemporaneously while only structural shocks that originate within the country affect inflation rates. This means that shocks in the foreign money supply, for example, cannot affect the domestic inflation rate *during* the period. Again, since we examine monthly data and since studies of the inflation processes within these countries typically show fairly long lags (*e.g.* Darby and Lothian *et al.*, 1983) this is not a very restrictive assumption either.

One way to assess the importance of temporary shocks is to calculate their contribution to the (conditional) variance of the real exchange rate. Their contribution is measured by the ratio

$$\langle 7 \rangle \qquad R_t = h_1 A_t^+(0)\, \Sigma_t\, A_t^+(0)' h_1' \left[h_1 A_t(0)\, \Sigma_t\, A_t(0)' h_1' \right]^{-1},$$

where $h_1 \equiv [1, 0, 0]$ and $A_t^+(0) = A_t(0)$ with $a_{3,t} = a_{4,t} = 0$. If the real exchange rate follows a random walk, all shocks are permanent and R_t will equal zero. If temporary shocks make some contribution to the variance of real exchange rates, $R_t > 0$. Notice also that this ratio can vary over time through changes in the covariance matrix Σ_t and the impact matrices $A_t^+(0)$ and $A_t(0)$. We will therefore be able to investigate whether the contribution of temporary shocks to the variance of real exchange rates has varied over the floating rate period.

Another way to examine the importance of temporary shocks is to calculate the hypothetical path of the real exchange rate under the assumption that all the shocks are temporary. Specifically, if estimates of the structural shocks U_t are available, the hypothetical path of the real exchange rate with only temporary shocks can be calculated from

$$\langle 8 \rangle \qquad \tilde{q}_t = q_{t-1} + h_1 A_t(L)[u_{1,t}, u^*_{1,t}, 0, 0]'.$$

The series for $\tilde{q}_t$ can then be compared against the actual real exchange rate to assess whether temporary shocks significantly contribute to the major movements in q_t.

These two ways of looking at the contribution of temporary shocks are complementary. The ratio R_t allows us to examine how temporary shocks contribute to the variability of the real exchange rate in the very short run. A comparison of the paths for q_t and $\tilde{q}_t$ reveals how the effects of temporary shocks cumulate over the medium run. (Temporary shocks, by definition, have no cumulative effect on the real exchange rate in the long run.)

II.B Econometric identification

Our decompositions of the real exchange rate require estimates of $A_t(L)$, Σ_t, and U_t. To obtain these estimates we make use of an alternative representation for the process in ⟨5⟩:

$$\langle 9\rangle \qquad \Delta X_{t+1} = B_t(L)V_{t+1}, \qquad B_t(0) = I, \qquad E_t V_{t+1} V'_{t+1} = \Omega_t.$$

Here V_t is a 3×1 vector of innovations to X_t which are related to the shocks U_t by

$$\langle 10\rangle \qquad V_{t+1} = A_t(0)U_{t+1}.$$

The time-varying matrix polynomial $B_t(L)$ is related to $A_t(L)$ in ⟨5⟩ in a complicated fashion and is described in the appendix.

We will now show how estimates of $B_t(L)$, V_t and Ω_t can be used to calculate the decompositions of the real exchange rate shown above. The procedure for estimating $B_t(L)$, V_t and Ω_t is described below.

The matrices $A_t(0)$ and Σ_t used to calculate the variance ratio R_t are found by solving three sets of equations. First, the representation in ⟨9⟩ and ⟨10⟩ implies

$$\langle 11\rangle \qquad \Omega_t = A_t(0)\,\Sigma_t\,A_t(0)'.$$

Since Ω_t is a 3×3 covariance matrix, this equation serves to identify 6 of the 9 unknown elements in $A_t(0)$ and Σ_t ($a_{1,t}, \ldots, a_{8,t}$, and ρ_t). The remaining elements are identified from intertemporal restrictions.

Substituting ⟨10⟩ into ⟨9⟩, we note that the long-run impact of the structural shocks U_t on X_t is given by $B_t(1)A_t(0) \equiv [\phi_{i,j,t}]$. Since the first two rows of U_t are the temporary shocks, their long-run impact on the level of the real exchange rate is measured by $\phi_{1,1,t}$ and $\phi_{1,2,t}$. By definition therefore,

$$\langle 12\rangle \qquad \phi_{1,1,t} = 0 \quad \text{and} \quad \phi_{1,2,t} = 0,\ \forall t.$$

Since the matrix $B_t(1)$ can be calculated from the estimate of $B_t(L)$, these requirements place two further restrictions in the matrix $A_t(0)$.

The final identifying restriction is a symmetry restriction. We assume that a permanent domestic shock, $u_{2,t}$, has the same *long-run* effect on the real exchange rate as a permanent foreign shock of the same magnitude (but opposite sign). This is a very natural restriction to impose because the real exchange rate is just a relative price. In the *long run* it should therefore just respond to *relative* productivity or taste shocks across countries (*i.e.* $u_{2,t} - u^*_{2,t}$ because q_t is the log real exchange rate). Since the third and fourth rows of U_t contain the permanent shocks, the symmetry restriction implies that

$$\langle 13\rangle \qquad \phi_{1,3,t} = \phi_{1,4,t}\ \forall t,$$

where $B_t(1)A_t(0) \equiv [\phi_{i,j,t}]$.

Together the restrictions in ⟨11⟩ to ⟨13⟩ are sufficient to identify all the elements in $A_t(0)$ and Σ_t. These matrices, in turn, are all that is required to calculate the variance ratio R_t.

To calculate the hypothetical path of the real exchange rate we need estimates of the four structural shocks, U_t. Equation ⟨10⟩ shows how these shocks are related to V_t, the 3×1 vector of innovations to exchange and inflation rates. Clearly the four structural shocks in U_t cannot be uniquely identified from the innovations, V_t, and the impact matrix, $A_t(0)$, without imposing a further restriction. We impose a neutrality restriction on the long-run effects of temporary shocks on the price level: the long-run effect of domestic temporary shocks on the domestic price level is restricted to be the same as the long-run effect of foreign temporary shocks on the foreign price level. Mathematically, this requires that

$$\phi_{2,1,t} = \phi_{3,2,t} \ \forall t. \tag{14}$$

In most models temporary shocks are purely nominal shocks, so neutrality implies the stronger condition that $\phi_{2,1,t} = \phi_{3,2,t} = 1$ for all t. We make use of the weaker condition in ⟨14⟩ because errors in measuring the appropriate price indices may invalidate the restrictions $\phi_{2,1,t} = 1$ and $\phi_{3,2,t} = 1$ in the data *even* when neutrality holds. Equation ⟨14⟩ allows for such measurement errors as long as they are equal across countries.

We use the restriction in ⟨14⟩ to identify the structural shocks U_t in the following way. First, we pre-multiply both sides of ⟨10⟩ by $h_2B_t(L)$ where $h_2 \equiv [0, 1, 0]$. This gives

$$h_2B_t(1)V_t = [\phi_{2,1,t}, \phi_{2,2,t}, \phi_{2,3,t}, \phi_{2,4,t}]U_t.$$

Next, we substitute the restriction in ⟨14⟩:

$$h_2B_t(1)V_t = [\phi_{3,2,t}, \phi_{2,2,t}, \phi_{2,3,t}, \phi_{2,4,t}]U_t. \tag{15}$$

⟨10⟩ and ⟨15⟩ provide us with four linearly independent equations which, given the estimates of V_t, $A_t(0)$ and $B_t(1)$, can be solved to find the vector of structural shocks. The structural shocks can then be used to construct the hypothetical path of the real exchange rate following the method described above.

II.C Estimation

The identification methods above require estimates of $B_t(1)$, Ω_t, V_t and $A_t(L)$. We obtain these estimates from a VAR:

$$C_t(L)\Delta X_t = Z_t \qquad \text{with } C_t(0) = I \ \forall t. \tag{16}$$

The VAR only approximates the process in ⟨9⟩ because the matrix polynomial $C_t(L)$ includes only a finite number of terms whereas $B_t(L)^{-1}$, the autoregressive polynomial implied by ⟨9⟩, may contain an infinite number. As a consequence, the innovations to the VAR may only approximately equal V_t, and $C_t(1)^{-1}$ may only approximate $B_t(1)$. However, the appendix shows that the approximation errors will be small when temporary shocks make relatively little contribution to the movements in X_t. As there is little evidence of mean reversion in real exchange rates, we believe that for our data these approximations are quite accurate.

To estimate the process in ⟨16⟩ we need to specify how the coefficients of the VAR vary through time. Unfortunately, without a detailed theoretical model of exchange rates and inflation it is impossible to provide a theoretical rationale for a particular dynamic process governing the variation in $C_t(L)$. Our specification for $C_t(L)$ is therefore chosen on empirical grounds. We estimated fixed coefficient VAR's of different orders, and tested for parameter instability to see whether there were any obvious shifts in the dynamics of X_t. Surprisingly, as shown below, we were unable to find any evidence of structural shifts when we included at least 4 lags in the VAR's. Thus, as an *empirical* matter, it appears that exchange rates and prices can be adequately represented by:

$$\langle 17 \rangle \qquad C(L)\Delta X_t = Z_t \qquad \text{with } C(0) = I \; \forall t,$$

where $C(L)$ has fixed coefficients. We use the estimate of $C(1)^{-1}$ to calculate $B(1)$, and the estimates of Z_t for V_t. The time varying covariance matrices, Ω_t, are obtained by estimating an ARCH process (specified below) for the residuals Z_t.

The hypothetical path for the real exchange rate requires an estimate of $A_t(L)$. In the absence of a detailed structural model, we use the approximation $A_t(L) \approx C(L)^{-1}A_t(0)$. This may be a poor approximation for some lags, L, because it constrains all the time variation in $A_t(L)$ to the impact matrix $A_t(0)$. However, the approximation holds exactly for $L = 0$, since $C(0) = \mathrm{I}$. At longer lags the appendix shows that the cumulative effects of temporary shocks may be exaggerated by the use of the approximation. In light of that, we need to exercise care in interpreting the hypothetical paths calculated for the real exchange rate.

III. Estimated real exchange-rate components

III.A Data and characteristics of the VARs

We use monthly data on nominal exchange and inflation rates from the *International Financial Statistics* database over the period 1975–1989. For the US, Italy and Japan, the monthly inflation rate is calculated from the index of wholesale prices. For the UK and Germany, they are calculated using the price index for industrial output and the wholesale industrial price index, respectively.

The implicit assumption behind $\langle 5 \rangle$ is that the first difference of the real exchange rate and the two inflation rates are appropriately modeled as a VAR. First we verified that Δq_t, Δp_t and Δp_t^* are indeed stationary processes. Next we checked for cointegration between the level of the real exchange rate q_t, and prices, p_t and p_t^*. The specification in $\langle 5 \rangle$ assumes that these series are not cointegrated. If they are, then $\langle 5 \rangle$ should be replaced by an error-correction representation. There is, however, no evidence of cointegration in our data.[11]

Table 2 reports the VAR specifications for each of the four countries. We used a series of Wald tests (corrected for heteroscedasticity) to decide on the appropriate number of lags to include in the VARs. Using the estimates from a VAR containing 12 lags, we conducted a series of Wald tests on the joint significance of Δq_{t-i}, Δp_{t-i} and Δp_{t-i}^* at lag i through 12, for $i \leqslant 12$. These tests showed that 6 lags were sufficient in all but the £/$ model, where the persistence of UK inflation requires that 12 lags be used. These models appear to capture all the serial correlation in the data. Table 2 shows that the residual correlations [$v_{\Delta q}$, $v_{\Delta p}$ and $v_{\Delta p^*}$] are all very small and none of the Q-statistics are statistically significant at the 5 per cent level.

The innovations from the VARs exhibit a good deal of heteroscedasticity. We model the conditional covariance matrix Ω_t as a multivariate GARCH(M,N)

TABLE 2. VAR summary statistics.

	VAR Lags	Ω_t Model		Residual Correlations ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	Q-stat[a]	Stability[b] (sign.)
DM/$	6	GARCH(1,1)	$v_{\Delta q}$	0.008	0.000	0.048	0.027	0.004	0.050	1.068	−0.085
			$v_{\Delta q}/\omega_{\Delta q}$	−0.110	0.018	0.010	−0.012	0.031	0.101	4.454	(0.534)
			$v_{\Delta p}$	−0.032	−0.076	−0.079	0.066	0.025	−0.027	3.567	−0.020
			$v_{\Delta p}/\omega_{\Delta p}$	0.017	−0.104	0.030	−0.029	−0.060	0.058	3.693	(0.508)
			$v_{\Delta p^*}$	0.056	0.029	−0.010	−0.003	0.069	−0.037	1.948	0.019
			$v_{\Delta p^*}/\omega_{\Delta p^*}$	−0.022	0.066	−0.076	−0.001	−0.031	0.204	9.939	(0.492)
¥/$	6	GARCH(1,2)	$v_{\Delta q}$	−0.005	0.020	0.026	−0.044	0.027	−0.001	0.715	−0.017
			$v_{\Delta q}/\omega_{\Delta q}$	−0.119	−0.038	−0.076	0.132	0.023	0.006	7.370	(0.507)
			$v_{\Delta p}$	0.025	0.094	0.144	0.070	0.042	0.063	7.810	0.002
			$v_{\Delta p}/\omega_{\Delta p}$	0.004	−0.121	0.064	0.143	0.024	0.107	9.619	(0.499)
			$v_{\Delta p^*}$	−0.036	−0.028	0.018	0.014	0.024	−0.067	1.468	−0.013
			$v_{\Delta p^*}/\omega_{\Delta p^*}$	0.185	−0.021	−0.006	0.026	−0.051	0.202	14.690*	(0.505)
£/$	12	GARCH(1,2)	$v_{\Delta q}$	0.001	0.002	−0.022	0.050	−0.016	−0.019	0.688	−0.014
			$v_{\Delta q}/\omega_{\Delta q}$	0.065	−0.017	0.122	0.081	−0.001	−0.021	4.950	(0.506)
			$v_{\Delta p}$	0.079	0.042	0.001	0.028	0.043	−0.020	2.093	0.026
			$v_{\Delta p}/\omega_{\Delta p}$	−0.045	0.020	−0.039	0.071	−0.005	−0.058	2.308	(0.489)
			$v_{\Delta p^*}$	−0.033	0.050	−0.045	0.043	0.006	−0.050	1.915	0.017
			$v_{\Delta p^*}/\omega_{\Delta p^*}$	−0.046	−0.009	−0.065	−0.040	0.084	0.027	2.953	(0.493)
Lit/$	6	GARCH(1,2)	$v_{\Delta q}$	0.007	0.023	−0.009	0.021	−0.039	−0.011	0.528	−0.060
			$v_{\Delta q}/\omega_{\Delta q}$	−0.087	−0.006	−0.055	−0.046	−0.041	0.013	2.728	(0.524)
			$v_{\Delta p}$	−0.052	−0.069	0.020	0.079	0.003	−0.070	3.620	0.032
			$v_{\Delta p}/\omega_{\Delta p}$	0.032	0.004	−0.066	−0.037	0.102	−0.023	3.293	(0.487)
			$v_{\Delta p^*}$	−0.001	0.039	−0.008	−0.048	−0.053	−0.050	1.743	0.005
			$v_{\Delta p^*}/\omega_{\Delta p^*}$	0.052	−0.029	−0.043	−0.054	−0.022	0.277	16.020*	(0.498)

Notes: [a]Box–Pierce Q statistics (6 lags) '*' indicates significance at 5% level.
[b]Recursive t-statistic for stability of each VAR equation.

process:

$$\Omega_t = \Omega_0 + \sum_{n=1}^{N} H_n V_{t-n} V'_{t-n} + \sum_{m=1}^{M} G_m \Omega_{t-m}. \tag{18}$$

To assure that Ω_t remains positive definite we estimated the Choleski factors of the matrices H and G.

The adequacy of the GARCH specifications can be judged from the correlations of the standardized squared VAR residuals $(v_{\Delta q}/\omega_{\Delta q})^2$, $(v_{\Delta p}/\omega_{\Delta p})^2$ and $(v_{\Delta p^*}/\omega_{\Delta p^*})^2$ where $\omega_{\Delta q}$, $\omega_{\Delta p}$ and $\omega_{\Delta p^*}$ are the estimated conditional standard deviations from the GARCH model. If the GARCH model captures all the conditional heteroscedasticity in the VAR residuals, these standardized residuals should be uncorrelated at all leads and lags (see Bollerslev, 1986). The majority of the correlations in Table 2 are very small and insignificantly different from zero at the 5 per cent level. There is, however, some marginal evidence of serial correlation in the US inflation residuals $[(v_{\Delta p^*}/\omega_{\Delta p^*})^2]$ in the ¥/\$ and Lit/\$ models. Experiments with higher order GARCH specifications showed that this minor misspecification of the Ω_t process did not affect the results reported below.

We also tested the VARs for structural stability. To do this we calculated a series of recursive residuals by repeatedly estimating the VARs over an ever expanding data sample. If there is no structural change, the sum of the recursive residuals should not differ significantly from zero. This hypothesis can be tested with a t-test (see Harvey, 1990, p. 157). These test statistics, reported in the right-hand column of Table 2, indicate no evidence of structural change.

As emphasized above, our statistical methodology uses the joint behavior of inflation and real exchange rates to identify the sources of exchange rate fluctuations. An indication of the usefulness of this approach can be gained from examining some simple implications of the VARs.

The right hand columns of Table 3 report tests for the joint significance of lagged Δp_t and Δp_t^* in the Δq_t equations of the VARs. The statistics show that lagged US inflation helps predict future changes in real exchange rates. This finding is broadly consistent with the idea that temporary shocks to real exchange rates also affect inflation rates.

The results reported in the center three columns of the table speak more directly to the usefulness of inflation in identifying the role of temporary shocks. Here we report the mean squared errors (MSE) of dynamic forecasts for Δq_t at 1, 2 and 3 month horizons generated from the VARs estimated over the whole sample period.[12] The first row of each block reports the MSEs calculated from a restricted VAR that excludes lagged inflation from the exchange rate equation. The MSEs in the second row use an unrestricted VAR that includes lagged inflation in the equation. The third row shows the percentage fall in the MSEs when the unrestricted rather than restricted VARs are used. In all cases, the percentage improvement is greatest at the one month horizon indicating that lagged inflation is more helpful in predicting future exchange rate movements in the near future. This is what we would expect to observe if fluctuations in inflation are informative about the role of temporary shocks in exchange rate movements.

III.B Overview of results

Figures 1 through 3 and Table 4 summarize the results from estimating our empirical model. Plotted in Figure 1 are the percentage contributions of the

TABLE 3. VAR forecast comparisons.

Currency	(Model)	In-Sample Mean Squared Forecast Errors Forecast Horizon (months) 1	2	3	Granger Causality Tests Δp (sign.)	Δp^* (sign.)
DM/$						
	(Without Inflation)	6.57	7.33	7.48	6.220	17.534
	(With Inflation)	6.08	7.13	7.36	(0.399)	(0.007)
	(Per cent Improvement)	8.05	2.89	1.64		
¥/$						
	(Without Inflation)	5.84	6.59	6.65	4.005	13.653
	(With Inflation)	5.30	6.05	6.20	(0.676)	(0.034)
	(Per cent Improvement)	10.25	9.00	7.22		
£/$						
	(Without Inflation)	5.66	6.81	6.84	16.339	20.011
	(With Inflation)	5.11	6.22	6.37	(0.176)	(0.067)
	(Per cent Improvement)	10.85	9.51	7.29		
Lit/$						
	(Without Inflation)	5.02	5.57	5.48	4.331	11.359
	(With Inflation)	4.74	5.45	5.35	(0.632)	(0.078)
	(Per cent Improvement)	6.05	2.27	2.33		

Notes: The Granger Causality test statistics indicate the significance of lagged inflation in the exchange rate equations of the VARs. The columns headed Δp and Δp^* refer to the significance of US inflation respectively. All test statistics are corrected for the presence of conditional heteroscedasticity.

temporary components to the month-to-month variations in the four real exchange rates, the R_t ratios defined in ⟨7⟩. Plotted in Figure 2 are the actual real exchange rates and the hypothetical paths that they would have followed had all shocks been temporary. These latter series show the cumulative contributions of the monthly temporary components plotted in Figure 1, and thus are medium-frequency analogues of those high-frequency measures. Figure 3 provides plots of the nominal exchange rates and their hypothetical paths due to temporary shocks similar to those plotted in Figure 2 for the real exchange rates. Table 4 reports the results of tests of significance of the monthly temporary components.

The picture painted by Figure 1 is clear in one very important respect: in no instance, are the contributions of the temporary shocks to the shorter term variations in real exchange rates zero, as they would be under the random walk hypothesis. In fact, for the DM/$ they average roughly 17 per cent of the total and at their peak reach 25 per cent. For the £/$ and the Lit/$ they average 5.8 per cent and 6.6 per cent, respectively, and run as high as 14 per cent and 15 per cent, respectively. For the ¥/$, they average 5.1 per cent and reach a high of 15 per cent.

In addition to these cross-currency differences in the average values of the R's, we also see pronounced differences in temporal patterns. For both the £/$ and

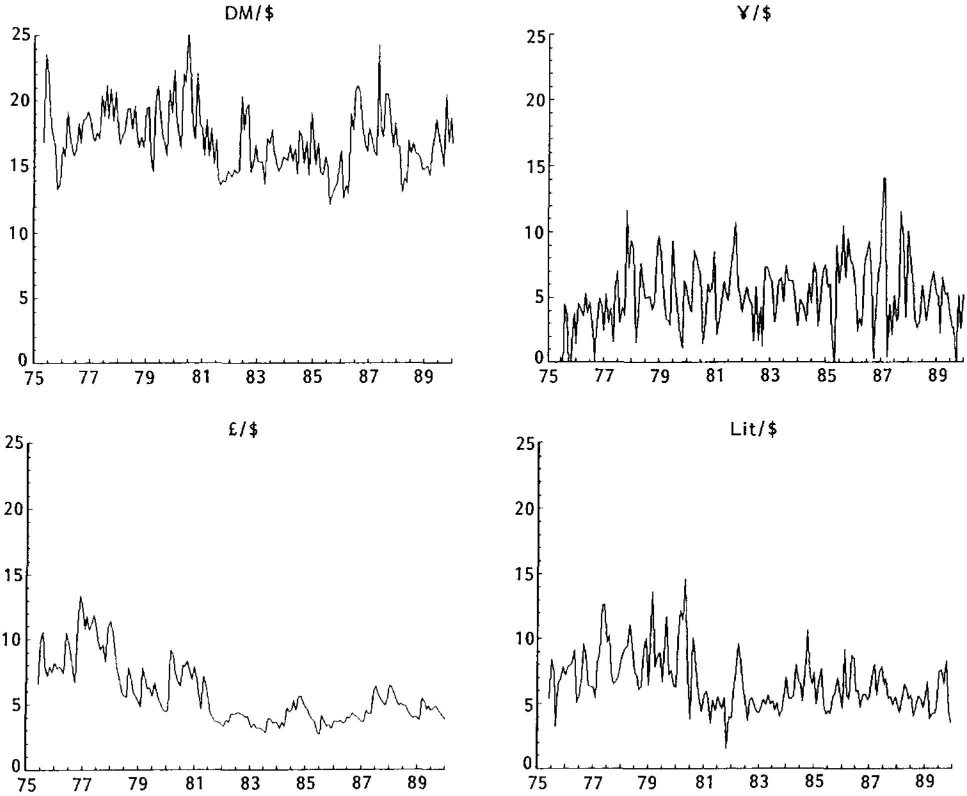

FIGURE 1. Percentage contribution of temporary shocks to the variance of (log) real exchange rates.

the Lit/\$, the R_ts show a marked secular decline between the 1970s and the 1980s, while for the DM/\$, R_t exhibits a noticeable cyclical pattern. By contrast, there are no persistent variations in R_t for the ¥/\$.

Figure 2 provides evidence on the cumulative effects of the monthly temporary components.[13] By comparing the hypothetical paths plotted there with the actual real exchange rates, we get an insight into both the importance of the temporary components at medium frequencies, and the degree to which their impact varies over time and across currencies. For the real Lit/\$ rate, these cumulative effects clearly are greatest, accounting for over 20 per cent of the variation over the full sample period. For the other three currency pairs, the cumulative effects of temporary shocks on average appear to be more heavily dominated by the effects of permanent shocks. In each instance, however, this is principally the result of the sharp divergence between the two series in the early and mid-1980s.[14]

Table 4 reports the results of significance tests of the temporary components and thus provides an important complement to the visual evidence presented in Figures 1 and 2. Listed in the table are the results of a series of t-tests for the null hypothesis $\phi = 0$, where ϕ is the slope coefficient in a regression of the monthly change in the real exchange rate Δq_t on the monthly change in the hypothetical path $\Delta \tilde{q}_t$:[15]

$$\Delta q_t = \phi \Delta \tilde{q}_t + e_t. \qquad \langle 19 \rangle$$

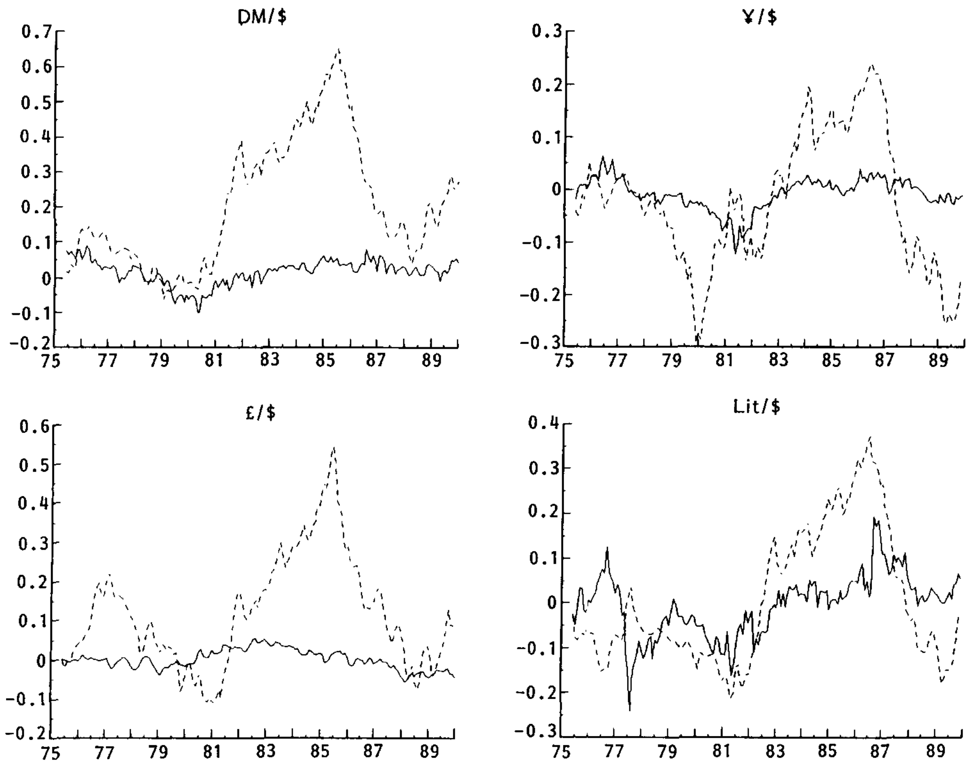

FIGURE 2. Hypothetical and actual paths for (log) real exchange rates. Path generated by temporary shocks ——, Actual Path -----.

We ran these regressions for each exchange rate separately over the full period and for overlapping five-year sub-periods. For all four currency pairs for at least part of the period, and for all but the Lit/$ for the full sample period, we are able to reject the null hypothesis of no relationship.

In the case of the Lit/$, the other evidence that we have just reviewed suggests that the cumulative effects of temporary shocks are substantial. Statistical significance, however, is largely confined to the period prior to 1982.

The plots of the nominal exchange rates and their hypothetical paths due to temporary shocks are shown in Figure 3. We calculated the latter by adding the hypothetical inflation differential to the hypothetical real exchange rate shown in Figure 2. The hypothetical inflation differential is propagated using the temporary shocks alone. The hypothetical nominal path can, therefore, be viewed as the path that the nominal exchange rate would have taken if both economies were just subject to shocks that had no long-run effect on the level of the real exchange rate.

In all four cases, the movements in the actual nominal and real exchange rates plotted in Figures 3 and 2 respectively mimic one another, which is exactly the pattern of correlation between the effective nominal and real rates described earlier in Table 1. Comparing the actual nominal rates with their hypothetical paths, we see a slightly closer relationship in Figure 3 for the DM/$ than we did

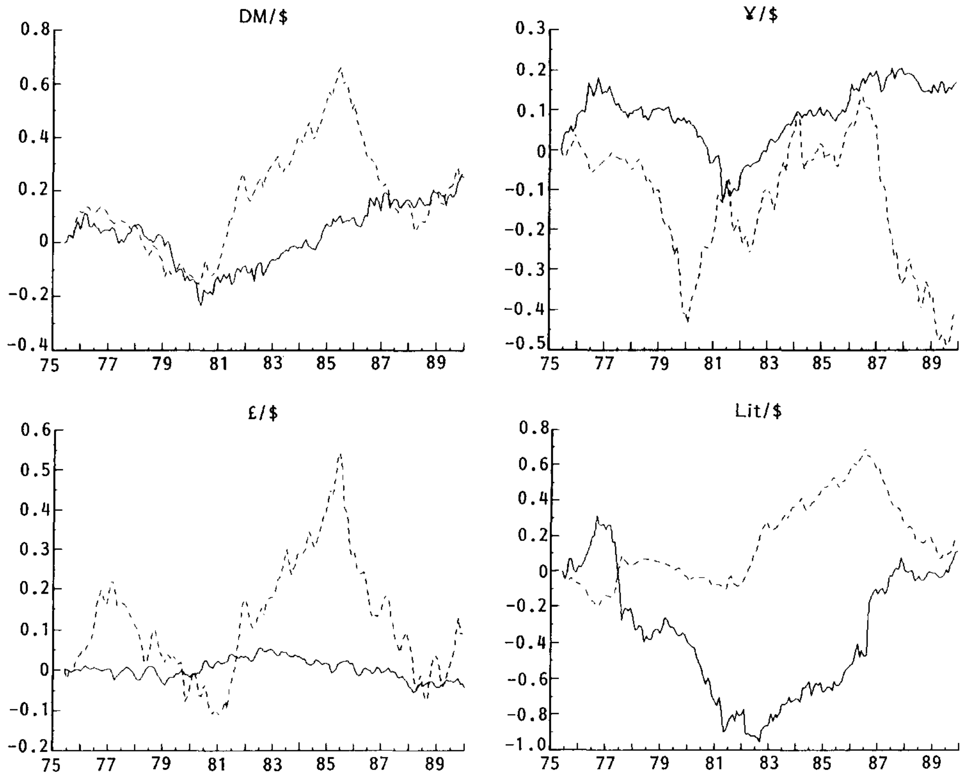

FIGURE 3. Hypothetical and actual paths for (log) nominal exchange rates. path generated by temporary shocks ——, Actual Path -----.

in Figure 2 for the real and roughly the same relationship for £/$ in the two charts. Fairly substantial differences are apparent, however, for ¥/$ and Lit/$.

III.C Implications

Two principal sets of implications emerge from this analysis. The first and most important concerns the time-series representation for the four real exchange rates. The second has to do with the sources of fluctuations in exchange rates under the float.

In all instances, we find evidence of temporary influences on real exchange rates, the extent of which varies across-countries, over time and with the frequency of the data. Some variations in real exchange rates, therefore, are amenable to models that attribute variations in real exchange rates to temporary shocks, like monetary shocks. The random walk characterization *in extremis* clearly is wrong.

Viewed in terms of the high frequency results summarized in Table 4 and Figure 1, the random walk characterization of real exchange rates, appears worst for the DM/$ and, not surprisingly perhaps, least inappropriate for the ¥/$.[16] These charts imply a sizable moving average component in Δq_t for the DM/$ and only a small component for the ¥/$. Such cross-currency differences suggest that US monetary shocks were not unilaterally responsible for the short-term

TABLE 4. T-tests for the importance of temporary disturbances to real exchange rates.

	Eqn. $\Delta q_t = \phi \Delta \tilde{q}_t + e_t$						T-test Ho: $\phi = 0$						
					Sample Periods (January to January)								
	75–89	75–78	76–79	77–80	78–81	79–82	80–83	81–84	82–85	83–86	84–87	85–88	86–89
DM/$	5.41	1.82	1.77	2.20	3.19	3.85	4.57	3.20	3.04	1.79	2.16	2.12	2.00
	5.80	1.66	1.50	2.42	4.15	5.28	5.46	3.73	3.76	1.92	2.25	2.21	2.30
¥/$	2.73	−0.76	0.38	−0.07	2.67	1.05	1.95	0.66	2.58	1.86	2.12	2.11	2.77
	2.41	−1.25	0.58	−0.06	2.17	0.80	1.62	0.58	2.77	1.84	2.01	2.05	3.03
£/$	4.06	−0.31	1.10	1.42	3.98	3.02	2.99	0.69	1.13	1.18	2.01	2.42	2.50
	4.37	−0.36	1.14	1.39	4.75	3.79	3.51	0.66	0.95	1.26	2.45	2.97	2.83
Lit/$	0.04	−2.87	−3.07	−2.82	2.01	1.12	1.56	0.70	0.08	−0.46	−0.06	1.08	1.19
	0.03	−2.60	−2.86	−2.28	2.19	1.22	1.72	0.80	0.08	−0.54	−0.08	1.29	1.26

Notes: q_t is the real exchange rate, $\tilde{q}_t$ is the hypothetical path of the real exchange rate produced by temporary shocks. The upper statistic allows for conditional heteroscedasticity. The lower statistic allows for conditional heteroscedasticity and MA(3) serially correlated errors.

volatility of real exchange rates. If they had been, then we would expect to see similar (average) values for R_t in all instances. The cross-currency differences in the R_t appear more consistent with differences in bilateral coordination of monetary policies. *Ceteris paribus*, an unexpected monetary contraction in the USA should have a larger impact on the real exchange rate if it coincides with an unanticipated easing, as opposed to contraction, abroad. This reasoning suggests that the lack of (high frequency) coordination between the Federal Reserve and the Bundesbank *could* be responsible for the high values for R_t in the case of the DM/$. Similarly, the secular declines in R_t for the £/$ and Lit/$ are consistent with greater co-ordination of policy during the course of the 1980s between the central banks of the UK, Italy, and the USA.

The hypothetical paths shown in Figure 2, as we have already stated, point to much larger cumulative effects of temporary shocks for the real Lit/$ rate than for the other three currency pairs, but nevertheless a striking similarity in the average patterns of the four real exchange rates. One common and rather substantial set of movements appears largely responsible. This is the much-discussed appreciation, then depreciation, of the dollar in the early and mid-1980s. Very little of these two major swings are explained by the cumulative effects of temporary shocks even in the case of the Lit/$ rate. Given that these swings are common to all four currency pairs, the implication is that they were driven mainly by US economic forces.

In the half decade prior to the dollar's substantial appreciation, and then later in the years following its dramatic decline, our hypothetical-path series do however track the actual series closely. The only exception, and that only partially, is the ¥/$. In the other three cases, the two swings have more or less canceled out. This near equality between the actual real rates and their hypothetical paths is not simply the result of the way that we have constructed the series: the hypothetical paths and the actual real exchange rates both are normalized to (log) zero in 1975, and therefore might be expected not to diverge greatly in the early part of the period, but there is no reason that they should return to track at the end.

The fact that they do so provides additional evidence consistent with mean reversion. But given that the pattern is observed so infrequently within the sample period, unit-root tests are unlikely to detect it. Were such offsetting swings to be repeated, the results of such tests quite likely would be different, as they generally are in the longer term time series.

In this sense, the picture of real exchange rate behavior under the float is not much different from the picture for the major currencies historically. This similarity, in turn, lends credence to the view that the failure to reject the random walk hypothesis with floating-rate, as opposed to long-term, data has to do with the difference in the temporal span of the data.

If that is the case, however, it is less than a totally satisfying explanation. Left unexplained is the nature of the shocks producing these large swings. They clearly are not the temporary shocks of conventional monetary models. That conclusion seems to us to be rather robust, since it in no way hinges upon a particular empirical formulation of such a model. Nor do they appear to be the productivity shocks of equilibrium models. To explain these two major swings in such terms, one would have to argue that a positive (relative) productivity shock in the United States was followed by a positive (relative) productivity shock of nearly equal magnitude in all four foreign countries at virtually the same time. One would have to argue further that, as a matter of happenstance, these common foreign shocks were all roughly equal in magnitude to the preceding one in the USA *and* one would have to explain why such a pattern had been the rule in other periods too.

The charts presented in Figure 3 showing nominal exchange rates and their hypothetical paths add to the evidence we have reviewed in several respects. Again, they highlight the importance of the 1980's swings in creating departures of the actual from the hypothetical. Again, we see eventual convergence of the two series in three of four cases. With the exception of £/$, however, the quarter-to-quarter relationship between the two series differs between this set of charts and those presented in Figure 2. This difference has to do with the dynamics of the inflation processes in the respective countries, in particular the degree of persistence of the temporary shocks to inflation rates. In cases in which there is a good deal of difference in persistence, temporary shocks can lead to substantial effects on the inflation differentials and thus to big differences in the hypothetical paths of the real and the nominal exchange rates.

As mentioned above, the hypothetical path can be viewed as the path that the nominal exchange rate would have taken if both economies were just subject to shocks that had no long-run effect on the level of the real exchange rate. The charts therefore give us an insight into the low-frequency effects of temporary shocks. For the DM/$, Lit/$ and ¥/$ rates, the degree of inflation persistence evidently varies enough across the respective countries to translate into greater lagged effects than in the medium-frequency paths shown in Figure 2. As we can see in the charts, the result for DM/$ is somewhat less divergence between actual and hypothetical path than exhibited in Figure 2. For Lit/$ the divergence is actually greater. For ¥/$, the picture is mixed, a closer relationship earlier on, but greater divergence at the end of the period.

IV. Conclusions

The empirical results that we have just reviewed provide an important insight into the time series properties of real exchange rates – whether in fact real exchange rates follow a random walk. Tests based on data for industrial countries over the past two decades of floating rates appeared to admit of only one answer, that they did. Those results, however, were totally at variance with the results of studies using longer time series, which in general have found real exchange rates to be stationary.

Our results lie somewhere in between those two extremes. In each instance, the data indicate the presence of both temporary and permanent influences on real exchange rates over the sample period. These temporary components, moreover, are statistically significant for at least part of the sample period for all four exchange rates. We conclude, based on these findings, that the random walk characterization, while a useful empirical description of the data under the float, is an unreliable touchstone for discriminating among alternative theoretical models. Buttressing this conclusion is one small, but highly intriguing, additional bit of evidence – the observed tendency for the swings in all four real dollar exchange rates during the early and mid-1980s to roughly cancel out.[17]

Appendix

This appendix begins by showing how to obtain the time series representation of ΔX_t in equations ⟨9⟩ and ⟨10⟩ from the process in ⟨5⟩. We then examine the approximations used to estimate the models and identify the hypothetical paths for the real and nominal exchange rates.

Any time linear series process like ⟨5⟩ can be written in state space form

⟨A1⟩ $$\Delta X_t = \Phi_t Y_t,$$

⟨A2⟩ $$Y_t = \Lambda_t Y_{t-1} + \Gamma_t U_t, \qquad E_{t-1} U_t U_t' = \Sigma_t.$$

⟨A1⟩ is the observation equation that relates the observed values for ΔX_t to the state vector, Y_t, governed by the dynamics of the state equation ⟨A2⟩. The state vector will generally include lagged ΔX_t's and lagged U_t's. The matrices Φ_t, Λ_t and Γ_t may be time varying reflecting the time variation in $A_t(L)$.

The representation in ⟨9⟩ is found by applying the Kalman Filter to the ⟨A1⟩ and ⟨A2⟩. The filtering equations are

⟨A3⟩ $$X_t = \Phi_t Y_{t/t-1} + V_t,$$

⟨A4⟩ $$Y_{t+1/t} = \Lambda_t [Y_{t/t-1} + K_t V_t],$$

⟨A5⟩ $$P_{t+1/t} = \Lambda_t [I - K_t \Phi_t] P_{t/t-1} \Lambda_t' + \Gamma_t \Sigma_t \Gamma_t',$$

and

⟨A6⟩ $$K_t = P_{t/t-1} \Phi_t' [\Phi_t P_{t/t-1} \Phi_t']^{-1}.$$

To find the MA representation we repeatedly substitute for $Y_{t/t-1}$ in ⟨A3⟩ using ⟨A4⟩–⟨A6⟩:

$$\begin{aligned} X_t &= V_t + \Phi_t \Lambda_{t-1} [Y_{t-1/t-2} + K_{t-1} V_{t-1}] \\ &= V_t + \Phi_t \Lambda_{t-1} K_{t-1} V_{t-1} + \Phi_t \Lambda_{t-1} Y_{t-1/t-2} \\ &= V_t + \Phi_t \Lambda_{t-1} K_{t-1} V_{t-1} + \Phi_t \Lambda_{t-1} \Lambda_{t-2} K_{t-2} V_{t-2} + \ldots\ldots \end{aligned}$$

⟨A7⟩ $$= B_t(L) V_t.$$

Since V_t are the innovations to the ΔX_t process given *all* past information, $V_t = A_t(0) U_t$.

We can illustrate why the Kalman Filter is needed to construct the MA representation in ⟨9⟩ and ⟨10⟩ with the aid of a simple example. Below we shall show that the basic insights gained from this example carry over into more complex models.

Let x_t be a scalar, and assume that

⟨A8⟩ $$x_t = u_{1,t} + n_t, \qquad n_t = n_{t-1} + u_{2,t}$$

with $\mathrm{var}(u_{1,t}) = \sigma_1^2$ and $\mathrm{var}(u_{2,t}) = \sigma_2^2$. The process in ⟨A8⟩ can be represented as

⟨A9⟩ $$\Delta x_t = A(L) U_t,$$

where $A(L) = (1-L, L)$, $U_t = [u_{1,t}, u_{2,t}]$. Suppose, instead of the Kalman Filter representation above, we used the standard Wold representation for Δx_t:

⟨A10⟩ $$\Delta x_t = v_t - \theta v_{t-1} = (1 - \theta L) v_t,$$

where $\theta = \sigma_1^2 / (\sigma_1^2 + \sigma_2^2)$. Since v_t are the innovations to x_t, $v_t = A(0) U_t$. It appears therefore that $\Delta x_t = (1-\theta L) A(0) U_t = A(L) U_t$ so $(1-\theta L) A(0) = A(L)$. However, if we write out this equation, $(1-\theta L)(1, 1) = (1-L, L)$, we see that it *only* holds true for $L = 0$.

This example shows that while the structural shocks U_t and the innovations to the standard Wold representation V_t are related by $V_t = A(0) U_t$, the dynamic lag structure in the Wold representation [*i.e.* $(1-\theta L)$ above] does not generally represent the dynamic lag structure of the structural shocks $A(L)$. The Kalman Filter representation in ⟨A7⟩ avoids this problem by changing the 'weighting' of past innovations through the K_t matrix.

We can also use this simple example to examine the approximations described in the text. First, note that

$$\Delta x_t = (1 - \theta L) v_t \qquad (1 - \theta L)^{-1} \Delta x_t = v_t,$$

so

⟨A11⟩ $$\Delta x_t = \sum_{i=1}^{\infty} \theta^i \Delta x_{t-i} + v_t.$$

This means that a finite order autoregressive representation for Δx_t does not exist when $\sigma_1^2>0$. However, the error in truncating the number of autoregressive lags will be smaller the closer θ is to zero, *i.e.* when σ_1^2/σ_2^2 is small.

We may also use this simple example to examine the approximaton $A_t(L)\simeq C(L)^{-1}A_t(0)$. First, note from ⟨A8⟩ and ⟨A9⟩ that $A_t(L)=[1-L, 1]$. Thus $A_t(0)$ is equal to [1, 1]. Since ⟨A10⟩ implies that $C(L)^{-1}=(1-\theta L)$, the approximation is $A_t(L)=[1-L, 1]\simeq(1-\theta L)[1, 1]$. Clearly the approximation holds exactly at lag zero. At lag one, however, the impact of a temporary shock should be -1, while the approximaton estimates it to be $-\theta$. The approximation therefore exaggerates the cumulative effects of temporary shocks since $A_t(1)=[0\ 1]$ while $C(1)A_t(0)=[1-\theta, 1-\theta]$ and $1-\theta>0$.

We now examine the impact this approximation has on the regression results presented in Table 4. Let $\Delta x_t=u_{1,t}-u_{1,t-1}$, and $\Delta\tilde{x}_t$ be the estimated change in the temporary component (based on the approximation discussed above). Table 4 reports the results of the regression

⟨A12⟩ $$\Delta x_t=\phi\Delta\tilde{x}_t+\varepsilon_t.$$

Least squares theory implies that $\phi=1+\theta(1-\theta)/(1+\theta^2)$. Since $\Delta x_t=\Delta x_t+u_{2,t}$ in this example, the coefficient is biased upwards whenever $0<\theta<1$. This suggests that *in the absence of the attenuation bias from measurement errors*, the reported coefficients in Table 4 overstate the influence of temporary shocks.

We can account for this bias by considering the standardized coefficients, or t-ratios. If Δx_t were used in the regression, the t-ratio would be

$$\hat{t}=\sqrt{(2\sigma_1^2/\sigma_2^2)}.$$

When Δx_t is the regressor,

$$\begin{aligned}\tilde{t}&=\sqrt{(\phi^2(1+\theta^2)\sigma_1^2/\mathrm{var}(\varepsilon))}\\&=\sqrt{((1+\theta)^2\sigma_1^2/(1+\theta^2)\mathrm{var}(\varepsilon))}\end{aligned}$$

Since the variance of (ε) is greater than σ_2^2, and $(1+\theta)^2/(1+\theta^2)<2$, $\tilde{t}$ is smaller than $\hat{t}$. Thus, the use of the approximation to find Δx_t places a downward bias on the t-ratios. Since the presence of measurement errors does not bias t-ratios, this example suggests that the significant ratios reported in Table 4 actually understate the influence of temporary shocks on the behaviour of real exchange rates.

Although these results were generated using the very simple model in ⟨A8⟩, similar effects will be present in more complex models, because those models can be written in a form similar to ⟨A8⟩. In particular, any multivariate model that includes both permanent shocks and temporary shocks (with arbitrary serial correlation) can be written in the extended state space form:

⟨A13⟩ $$\begin{gathered}Y_t=\Lambda Y_{t-1}+\Gamma_1 U_{1,t}+\Gamma_2 N_t,\\ \Delta X_t=\Phi Y_t,\\ N_t=N_{t-1}+U_{2,t},\\ E_{t-1}U_{2,t}U'_{2,t}=\Sigma_2,\qquad E_{t-1}U_{1,t}U'_{1,t}=\Sigma_1,\end{gathered}$$

It is straightforward to show that the process for Y_t can also be represented by

⟨A14⟩ $$\Delta Y_t=\Lambda\Delta Y_{t-1}+V_t+\Theta V_{t-1},\quad \text{where } \Theta=-\Gamma_1\Sigma_1\Gamma'_1(\Gamma_1\Sigma_1\Gamma'_1+\Gamma_2\Sigma_2\Gamma'_2)^{-1}.$$

⟨A13⟩ and ⟨A14⟩ are multivariate analogous (incorporating arbitrary degrees of serial correlation) to the representations in ⟨A8⟩ and ⟨A9⟩ used as our example.

Notes

1. We discuss these studies in the next section of this paper. See Dornbusch (1987) for an overview of the state of knowledge about PPP, Frankel and Meese (1987) and Dornbusch (1988) for similar surveys dealing with real-exchange-rate behavior, and Meese (1990) for a discussion of the performance of monetary and portfolio balance models under floating rates.
2. See Lothian (1986) and Koedijk and Schotman (1989) for earlier descriptions of real dollar exchange rate movements under the float.
3. A notable exception is Kravis and Lipsey (1988).
4. Examples include Stockman (1980) and Kravis and Lipsey (1988).

5. See, for example, Enders (1988), Mark (1990) and Taylor (1988). Abuaf and Jorion (1990), who test the unit root hypothesis for a number of exchange rates jointly, do, however, present marginal evidence against it. In addition, see Cumby and Obstfeld (1984), Huizinga (1987), and Liu and He (1991), who also present results based upon floating-rate data that are inconsistent with the random walk hypothesis.
6. Prominent among the explanations advanced to date have been fiscal policy in the USA (Feldstein, 1986); the shift in the US monetary-policy regime (Lothian, 1986), rational learning in the face of that regime shift (Lewis, 1989), productivity shocks (Stockman, 1988), fad-like behavior (Frankel and Froot, 1987), changes in the real return to real investment in the USA and shifts in the relative demand for dollar-denominated assets (Dooley and Isard, 1991).
7. The long-term time-series investigations include Frankel (1986), Edison (1987), Enders (1989), Abuaf and Jorion (1990), Diebold *et al.* (1991), Kim (1990), Lothian (1990) and Lothian and Taylor (1992).
8. Examples include McNown and Wallace (1989) for several high inflation Latin American countries in the 1980s and Taylor and McMahon (1988) and Phylaktis (1992) for a number of countries in the 1920s.
9. Frankel (1986), Huizinga (1987) and Lothian (1990) have all made such an argument.
10. Hakkio and Rush (1991) provide Monte Carlo results supporting this conjecture.
11. The results of the unit root and cointegration tests are available upon request.
12. The mean squared error of the forecasts at horizon k is calculated as

$$(T-k)^{-1}\sum_{t=1}^{T-k}(\Delta q_{t+k}-\Delta q_{t+k/t})^2,$$

where $\Delta q_{t+k/t}$ is the forecast of Δq_{t+k} given information at time t generated from a VAR with the same lag structure as those in Table 2.
13. The differences between Figures 1 and 2 in the relative contributions of temporary shocks are explicable in terms of differences in the serial correlation in the adjustment to temporary shocks. When hit by temporary shocks, the DM/$ displays a good deal of negative serial correlation. Otherwise, the large value for R_t seen in Figure 1 would suggest large cumulative swings in the hypothetical path. Similar reasoning indicates a very different serial correlation structure for the Lit/$, since the cumulative movements are large relative to the average value of R_t.
14. Using Beveridge and Nelson's technique, Cumby and Huizinga (1991) decompose real exchange rates into permanent and temporary components. They find that while most of the movements in real exchange rates can be attributed to the permanent components, at times temporary components have a sizable influence. This accords with the findings of Huizinga (1987).
15. The coefficients in these regressions are subject to two sorts of opposing bias: a downward attenuation bias due to measurement error and an upward bias due to the procedure that we employ to identify the temporary component. The t values, as we show in the appendix, are biased downward.
16. Marston (1986) and Yoshikawa (1990) in their analyses of the ¥/$ exchange rate under the float attribute permanent shifts to faster productivity growth in Japan than in the USA. Lothian (1990) presents additional evidence on this phenomenon, showing the ¥/$ and ¥/£ are somewhat better described as trend stationary than stationary over the period from 1874 to 1989.
17. This is a broader based phenomenon. Lothian (1993), using cross-country averages for 22 OECD countries over the period 1974–1990 presents scatters of real changes in exchange rates relative to their nominal-rate and price-level components and of the one component relative to the other. These scatters show no relation between real-exchange-rate changes and either of these two components, and hence a new one-to-one relation between the one component and the other.

References

ABUAF, NISO AND PHILLIPE JORION, 'Purchasing Power Parity in the Long Run,' *Journal of Finance*, March 1990, **45**: 157–174.

ADLER, MICHAEL AND BRUCE LEHMAN, 'Deviations from Purchasing Power Parity in the Long Run,' *Journal of Finance*, December 1983, **38**: 1471–1487.

BLANCHARD, OLIVIER AND DANNY QUAH, 'The Dynamic Effects of Aggregate Demand Disturbances,' *American Economic Review*, September 1989, **79**: 655–673.

BOLLERSLEV, TIMOTHY, 'Generalized Autoregressive Conditional Heteroskedasticity,' *Journal of Econometrics*, April 1986, **31**: 307–327.

CUMBY, ROBERT AND JOHN HUIZINGA, 'The Predictability of Real Exchange Rate Changes in the Short and Long Run,' *Japan and the World Economy*, 1991, **3**: 17–38.

CUMBY, ROBERT AND MAURICE OBSTFELD, 'International Interest-Rate and Price-Level Linkages Under Flexible Exchange Rates: A Review of Recent Evidence,' in John Bilson and Richard Marston, eds, *Exchange Rate Theory and Practice*, Chicago: University of Chicago of the NBER, 1984.

DARBY, MICHAEL R. 'Movements in Purchasing Power Parity: The Short and Long Runs,' in Michael R. Darby, James R. Lothian, *et al.*, *The International Transmission of Inflation*, Chicago: University of Chicago Press for the NBER, 1983.

DARBY, MICHAEL R., JAMES R. LOTHIAN, AND ARTHUR E. GANDOLFI, ANNA J. SCHWARTZ AND ALAN C. STOCKMAN, *The International Transmission of Inflation*, Chicago: University of Chicago Press for the NBER, 1983.

DIEBOLD, FRANCIS X., STEPHEN HUSTED AND MARK RUSH, 'Real Exchange Rates Under The Gold Standard,' *Journal of Political Economy*, December 1991, **99**: 1252–1271.

DOOLEY, MICHAEL AND PETER ISARD, 'A Note on Fiscal Policy, Investment Location Decisions and Exchange Rates,' *Journal of International Money and Finance*, March 1991, **10**: 161–168.

DORNBUSCH, RUDIGER, 'Expectations and Exchange Rate Dynamics,' *Journal of Political Economy*, December 1976, **84**: 1161–1176.

DORNBUSCH, RUDIGER, 'Purchasing Power Parity,' in John Eatwell, Murray Milgate and Peter Newman, eds, *The New Palgrave: A Dictionary of Economics*, Stockton: New York, NY, 1987.

DORNBUSCH, RUDIGER, 'Real Exchange Rates and Macroeconomics: A Selective Survey,' NBER working paper 2775, 1988.

EDISON, HALI J., 'Purchasing Power Parity in the Long Run: A Test of the Dollar-Pound Exchange Rate, 1890–1978,' *Journal of Money, Credit and Banking*, August 1987, **19**: 376–387.

ENDERS, WALTER, 'ARIMA and Cointegration Tests of Purchasing Power Parity,' *Review of Economics and Statistics*, August 1988, **70**: 504–508.

ENDERS, WALTER, 'Unit Roots and the Real Exchange Rate Before World War I: The Case of Britain and the United States,' *Journal of International Money and Finance*, March 1989, **8**: 59–74.

EVANS, MARTIN, 'On the Changing Nature of the Output-Inflation Trade-Off,' Stern School of Business, New York University working paper, 1991.

FELDSTEIN, MARTIN, 'The Budget Deficit and the Dollar,' *NBER Macroeconomics Annual* 1986.

FRANKEL, JEFFREY A., 'International Capital Mobility and Crowding Out in the U.S. Economy: Imperfect Integration of Financial Markets or Goods Markets,' in R. Hafer, ed, *How Open is the U.S. Economy?*, Lexington, MA: Lexington Books, 1986.

FRANKEL, JEFFREY A. AND KENNETH A. FROOT, 'Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations,' *American Economic Review*, March 1987, **77**: 133–153.

FRANKEL, JEFFREY A. AND RICHARD MEESE, 'Are Exchange Rates Excessively Variable?,' in Stanley Fischer, ed, *NBER Macroeconomics Annual*, Cambridge, MA: MIT Press, 1987.

HAKKIO, CRAIG AND MARK RUSH, 'Cointegration: How Short is the Long Run?,' *Journal of International Money and Finance*, December 1991, **10**: 571–581.

HARVEY, A.C., 'The Econometric Analysis of Time Series,' Cambridge, MA: MIT Press, 1990.

HUIZINGA, JOHN, 'An Empirical Investigation of the Long-Run Behavior of Real Exchange Rates,' in Karl Brunner and Allan H. Meltzer, eds, *Carnegie-Rochester Conference Series on Public Policy 27*, North Holland: Amsterdam, 1987.

INTERNATIONAL MONETARY FUND, *International Financial Statistics*, various issues.

KIM, YOONBAI, 'Purchasing Power Parity in the Long Run: A Cointegration Approach,' *Journal of Money, Credit, and Banking*, November 1990, **22**: 348–357.

KOEDIJK, KEES G. AND PETER C. SCHOTMAN, 'Dominant Real Exchange Rate Movements,' *Journal of International Money and Finance*, December 1989, **8**: 517–531.

KRAVIS, IRVING B. AND ROBERT E. LIPSEY, 'National Price Levels and the Prices of Tradables and Non-Tradables,' *American Economic Review*, May 1988, **78**: 474–479.

LEWIS, KAREN K., 'Changing Beliefs and Systematic Rational Forecast Errors With Evidence From Foreign Exchange,' *American Economic Review*, September 1989, **79**: 621–637.

LIU, CHRISTINA Y. AND JIA HE, 'A Variance Ratio Test of Random Walks in Foreign Exchange,' *Journal of Finance*, June 1991, **46**: 777–786.

LOTHIAN, JAMES R., 'Real Dollar Exchange Rates Under the Bretton-Woods and Floating-Rate Systems,' *Journal of International Money and Finance*, December 1986, **5**: 429–448.

LOTHIAN, JAMES R., 'A Century Plus of Japanese Exchange Rate Behavior,' *Japan and the World Economy*, January 1990, **2**: 47–70.

LOTHIAN, JAMES R. AND MARK P. TAYLOR, 'The Float from the Perspective of the Last Two Centuries,' International Monetary Fund working paper, November 1992.

LOTHIAN, JAMES R., 'Multi-Country Evidence on the Behavior of Purchasing Power Parity under the Current Float,' Fordham University graduate school of Business Administration, working paper, June 1993.

LUCAS, ROBERT E. JR., 'Interest Rates and Prices in a Two-Currency World,' *Journal of Monetary Economics*, November 1982, **10**: 335–360.

MCNOWN, ROBERT AND MYLES S. WALLACE, 'National Price Levels, Purchasing Power Parity and Cointegration: A Test of Four High Inflation Economies,' *Journal of International Money and Finance*, December 1989, **8**: 533–545.

MARK, NELSON, 'Real and Nominal Exchange Rates in the Long Run: An Empirical Investigation,' *Journal of International Economics*, February 1990, **28**: 115–136.

MARSTON, RICHARD, 'Real Exchange Rates and Productivity Growth in the United States and Japan,' NBER working paper no. 1922, 1986.

MEESE, RICHARD, 'Currency Fluctuations in the Post-Bretton Woods Period,' *Journal of Economic Perspectives*, Winter 1990, **4**: 117–134

MEESE, RICHARD A. AND KENNETH ROGOFF, 'Empirical Exchange Rate Models of the Seventies: Do they Fit out of Sample?,' *International Economic Review*, February 1983, **14**: 3–24.

MEESE, RICHARD A. AND KENNETH ROGOFF, 'Was It Real?: The Exchange Rate-Interest Differential Relation over the Modern Floating Rate Period,' *Journal of Finance*, September 1988, **43**: 933–948.

MUSSA, MICHAEL, 'A Model of Exchange Rate Dynamics,' *Journal of Political Economy*, February 1982, **90**: 74–104.

PAGAN, ADRIAN, 'Econometric Issues in the Analysis of Regressions with Generated Regressions,' *International Economic Review*, February 1984, **21**: 221–247.

PHYLAKTIS, KATE, 'Purchasing Power Parity and Cointegration: The Greek Evidence from the 1920s,' *Journal of International Money and Finance*, October 1992, **11**: 502–513.

ROLL, RICHARD, 'Violations of Purchasing Power Parity and their Implications for Efficient International Commodity Markets,' in Marshall Sarnat and G.P. Szego, eds, *International Finance and Trade*, vol. 1, Cambridge, MA: Ballinger, 1979.

SHAPIRO, MATTHEW AND MARK WATSON, 'Sources of Business Fluctuations,' in Stanley Fisher, ed, *NBER Macroeconomics Annual*, Cambridge, MA: MIT Press, 1988.

STOCKMAN, ALAN C., 'A Theory of Exchange Rate Determination,' *Journal of Political Economy*, August 1980, **88**: 673–698.

STOCKMAN, ALAN C., 'Real Exchange Rate Variability Under Pegged and Floating Nominal Exchange Rate Systems: An Equilibrium Theory,' University of Rochester, Working paper no. 128, 1988.

TAYLOR, MARK P., 'An Empirical Examination of Long-Run Purchasing Power Parity using Cointegration Techniques,' *Applied Economics*, October 1988, **20**: 1369–1381.

TAYLOR, MARK P. AND P.C. MCMAHON, 'Long-Run Purchasing Power Parity in the 1920s,' *European Economic Review*, January 1988, **32**.

YOSHIKAWA, HIROSHI, 'On the Equilibrium Yen-Dollar Rate,' *American Economic Review*, June 1990, **3**: 576–583.