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REAL EXCHANGE RATES OVER THE PAST TWO CENTURIES: HOW IMPORTANT IS THE HARROD-BALASSA-SAMUELSON EFFECT?*

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Using data since 1820 for the US, the UK and France, we test for the presence of real effects on the equilibrium real exchange rate (the Harrod-Balassa-Samuelson, HBS effect) in an explicitly non-linear framework and allowing for shifts in real exchange rate volatility across nominal regimes. A statistically significant HBS effect for sterling–dollar captures its long-run trend and explains a proportion of variation in changes in the real rate that is proportional to the time horizon of the change. There is significant evidence of nonlinear reversion towards long-run equilibrium and downwards shifts in volatility during fixed nominal exchange rate regimes.

In this article, we investigate the influence of productivity differentials on the equilibrium level of the real exchange rate and the speed at which the real exchange rate converges towards that equilibrium.

Given that the real exchange rate is defined as the ratio of national prices expressed in a common currency, evidence of a long-run stable mean for the real exchange rate is a necessary condition for long-run purchasing power parity (PPP) to hold. The issue of whether or not the real exchange rate between major economies tends to revert towards a stable long-run equilibrium has been a topic of considerable debate in the literature (Taylor and Taylor, 2004).[1] Taylor *et al.* (2001) argue that the key both to detecting significant mean reversion in the real exchange rate and to finding speeds of mean reversion that are not 'glacially slow' (the 'PPP puzzle': Rogoff, 1996) lies in allowing for nonlinearities in real exchange rate adjustment, so that as the real exchange rate deviates from its long-run equilibrium, the forces driving it back towards equilibrium increase in strength more than proportionately.[2]

Parallel to the recent literature on nonlinearities in real exchange rate adjustment, researchers have also stressed the importance of real shocks to the underlying equilibrium real exchange rate. The idea that productivity shocks may affect the equilibrium real exchange rate – the so-called Harrod-Balassa-Samuelson (HBS) effect – has a fairly long history in economics (Froot and Rogoff, 1995; Taylor and Taylor, 2004). In general, this research provides mixed results. Earlier studies, with the exception of Lothian's (1990) investigation of Japanese exchange-rate behaviour, find virtually no

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[1] Early studies of mean reversion in real exchange rates include Taylor (1988) and Mark (1990).

[2] The cause of this nonlinearity may be greater goods arbitrage as the misalignment grows or a growing degree of consensus concerning the appropriate or likely direction of movements in the nominal exchange rate among traders, or perhaps a greater likelihood of the occurrence and success of intervention by the authorities to correct a strongly misaligned exchange rate. See Taylor and Taylor (2004) or Taylor (2006).

evidence of HBS effects. Later studies, for the most part, find only weak supporting evidence. A key point here is that if the equilibrium exchange rate is moving gradually over time but statistical tests for real exchange rate stability assume that the equilibrium exchange rate is constant, then estimates of the speed of reversion towards the mean will be biased, and this bias may be at least partly responsible for Rogoff's PPP puzzle. Evidence suggestive of a bias arising from this source is provided by studies which have found that allowing for linear or nonlinear deterministic trends (which may be proxying for HBS effects) can make a material difference in resolving the puzzles about whether and how fast the exchange rate moves to its PPP level (Taylor, 2002; Lothian and Taylor, 2000).

Our main contributions to this literature in the research reported in this paper are two-fold. First, we show that the HBS effect is important and emprically supported in a nonlinear setting, at least for the US–UK real exchange rate. Second, we explicitly model volatility shifts across nominal exchange rate regimes in an analysis of long-span data. We carry out an empirical analysis of real exchange rates and productivity differentials within a nonlinear framework, using a data set for the US, the UK and France covering the period 1820–2001 (1820–1998 for investigations involving the franc). We proxy the level of productivity by real GDP *per capita,* which allows us to examine the HBS effect using a long-span of data over which productivity differentials would be expected to be important even between major economies.

The remainder of the article is set out as follows. In the next Section we discuss methods for modelling nonlinearity in real exchange rate adjustment, while in Section 2 we briefly outline the theoretical rationale for the influence of productivity differentials on the long-run equilibrium real exchange rate. In Section 3 we discuss the evidence of shifting real exchange rate volatility across nominal exchange regimes and outline our empirical methods for allowing for these shifts. In Section 4 we describe our data set and in Section 5 we present our main empirical results. Section 6 contains the results of some exercises designed to gauge the importance of the HBS effect in the UK-US real exchange rate at various time horizons and in explaining the empirical significance of nonlinear trends in real exchange rate movements. We provide some concluding comments and suggestions for future research in a final Section.

1. Modelling Nonlinearity

A number of authors have reported evidence of nonlinearity in real exchange rate adjustment (Taylor, 2006). One particular statistical characterisation of nonlinear adjustment, which appears to work well for exchange rates, is the exponential smooth transition autoregressive (ESTAR) model (Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998; van Dijk *et al.*, 2002).[3] In the ESTAR model, adjustment takes place in every period but the speed of adjustment towards the long-run mean varies with the extent of the deviation from the mean. An ESTAR model for a time series process $\{y_t\}$may be written:

[3] For applications of the ESTAR model to exchange rates, see, e.g., Taylor *et al.* (2001), Taylor *et al.* (2001) and Kilian and Taylor (2003).

$$(y_t - \mu_0) = \sum_{j=1}^{p} \beta_j (y_{t-j} - \mu_0) + \left[\sum_{j=1}^{p} \beta_j^* (y_{t-j} - \mu_0)\right] \left\{1 - \exp[-\theta(y_{t-d} - \mu_0)^2]\right\} + \varepsilon_t \tag{1}$$

where $\varepsilon_t \sim \mathrm{N}(0, \sigma_\mathrm{t}^2)$, $\theta \in (0, +\infty)$ and μ denotes the mean or long-run equilibrium of the process. The exponential term $\{1 - \exp[-\theta(y_{t-d} - \mu_0)^2]\}$, a symmetrically inverse–bell-shaped function, is termed the transition function since it can be thought of as smoothly determining the transition of the autoregressive process between two extreme regimes, an inner regime and an outer regime. The inner regime corresponds to $y_{t-d} = \mu_0$, when the transition function vanishes and (1) becomes a linear AR(p) model:

$$(y_t - \mu_0) = \sum_{j=1}^{p} \beta_j (y_{t-j} - \mu_0) + \varepsilon_t. \tag{2}$$

The outer regime corresponds, for given θ, to $\lim_{|y_{t-d-\mu 0}| \to \infty} \{1 - \exp[-\theta (y_{t-d} - \mu_0)^2]\} = 1$, where (1) becomes a different AR(p) model:

$$(y_t - \mu_0) = \sum_{j=1}^{p} (\beta_j + \beta_j^*)(y_{t-j} - \mu_0) + \varepsilon_t \tag{3}$$

with a correspondingly different speed of mean reversion so long as $\beta_j^* \neq 0$ for at least one value of j.

In any particular application of the ESTAR model, of course, the parameters p and d must be chosen, and a number of selection procedures have been suggested in the literature (Lundbergh *et al.*, 2003). In the present context, economic intuition suggests a presumption in favour of smaller values of the delay parameter d rather than larger values, in that it is hard to imagine why there should be very long lags before the real exchange rate begins to adjust in response to a shock, especially where one is using annual data. In the research reported below, we used the model procedure suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994). This involves first choosing the order of the autoregression, p, by an examination of the partial autocorrelation function of the series and then estimating an equation similar in form to (1) but with the second term on the right-hand side replaced with cross products of y_{t-j} and first, second and third powers in y_{t-d}, for various values of d. This can be interpreted as a third-order Taylor series expansion of (1). The resulting equation is nonlinear in some of the variables but is linear in the parameters and so can be estimated by ordinary least squares; a test of the exclusion restrictions on the power and cross-product terms in this estimated equation is then a test for linearity against a linear alternative. The value of d is then chosen as that which gives the largest value of this test statistic.In the Monte Carlo study of Teräsvirta (1994), this selection procedure was shown to work well in terms of choosing the correct value of the delay parameter.

ESTAR models of the form (1) have been successfully applied to real exchange rates by, among others, Taylor *et al.* (2001) and Kilian and Taylor (2003), who effectively impose a constant value of the long-run equilibrium real exchange rate. In the analysis presented below, we extend this framework by introducing a potentially time-varying equilibrium value of the real exchange rate in order to allow for HBS effects. This can be analysed in the above framework by setting $(y_t - \mu_0) = (q_t - \mu_t)$ in (1), where q_t is the real exchange rate and μ_t is its time-varying equilibrium, so that the nonlinear ESTAR model employed in our investigation becomes:

$$(q_t - \mu_t) = \sum_{j=1}^{p} \beta_j (q_{t-j} - \mu_{t-j}) + \left[\sum_{j=1}^{p} \beta_j^* (q_{t-j} - \mu_{t-j})\right] \left\{1 - \exp[-\theta(q_{t-d} - \mu_{t-d})^2]\right\} + \varepsilon_t. \tag{4}$$

Our empirical specification for the time-varying equilibrium real exchange rate μ_t is discussed in the next Section.

Further, we also allow for shifts in variance in the error term $\{\varepsilon_t\}$, rather than assuming homoscedasticity as in previous studies of nonlinearity in real exchange rate movements.[4] As discussed above, this seems particularly appropriate since our data span a number of exchange rate regimes. The empirical specification for the residual variance is discussed in Section 4.

2. Productivity Differentials and Long-Run Equilibrium Real Exchange Rates

According to the HBS framework (Harrod, 1933; Balassa, 1964; Samuelson, 1964), a country experiencing relatively high productivity growth will find that its exchange rate tends to return to a level where its currency is overvalued on PPP considerations, and that the apparent degree of overvaluation on PPP grounds increases with the size of the differential in productivity between the home and foreign economies.

Suppose a country experiences productivity growth primarily in its traded goods sector and that the law of one price (LOP) holds among traded goods in the long run. Productivity growth in the traded goods sector will lead to wage rises in that sector without the necessity for price rises but workers in the non-traded goods sector will also demand comparable pay rises and this will lead to a rise in the price of non-tradables and hence a rise in the overall price index. Since the LOP holds among traded goods and, by assumption, the nominal exchange rate has remained constant, this means that the upward movement in the home price index will not be matched by a movement in the nominal exchange rate so that, if PPP initially held, the home currency must now appear overvalued on the basis of comparisons made using price indices expressed in a common currency at the prevailing nominal exchange rate. The crucial assumption is that productivity growth is higher in the traded goods sector.

We can analyse this issue more formally as follows. Consider an economy ('Home') that has two sectors, one producing a composite tradable good and one producing a composite non-tradable good. Assuming Cobb-Douglas preferences yields a consumption-based price index that is a geometric weighted average of the Home prices of tradables and non-tradables:

$$P \equiv P_T^{\gamma} P_N^{1-\gamma}, \tag{5}$$

where P_N and P_T denote the price of non-tradables and tradables, respectively, P is the consumer price index and γ $(0 < \gamma < 1)$ is a constant parameter. In the long run, labour is perfectly mobile between sectors so that workers receive the same long-run real wage in each sector, i.e. $W_T/P = W_N/P$, where W_T and W_N represent the nominal

[4] An exception to this is the recent study by Paya and Peel (2006).

wage in the tradable and non-tradable sectors, respectively. Therefore, the nominal wage is also equalised across sectors in the long run: $W_T = W_N = W$, say. However, firms in each sector pay a long-run nominal wage that is equal to the marginal revenue product of labour in that sector, i.e. $W_T = W = P_T A_T$ and $W_N = W = P_N A_N$, where A_N and A_T denote the marginal product of labour in the tradable and non-tradable sectors respectively. Combining this with (5), we have:

$$P = P_T (A_T / A_N)^{(1-\gamma)}. \tag{6}$$

Equation (6) encapsulates the HBS condition that relatively higher productivity growth in the tradables sector will tend to generate a long-run rise in the relative price of non-tradables and hence a rise in the overall price level. This translates into an appreciation of the real exchange rate through the law of one price, which is expected to hold among tradable goods in the long run:

$$P_T^* = P_T S, \tag{7}$$

where an asterisk (here and below) denotes a variable or coefficient in the trading partner economy ('Foreign') or a Foreign coefficient and S is the exchange rate (the Foreign price of Home currency). Assuming that equations similar to (5) and (6) hold for the Foreign economy, the following expression for the long-run equilibrium real exchange rate, Q, can be derived

$$Q \equiv SP/P^* = (A_T/A_N)^{(1-\gamma)} / (A_T^*/A_N^*)^{(1-\gamma^*)}. \tag{8}$$

If the composition of consumption in terms of tradable and non-tradable goods is similar in both countries (i.e. γ is close to γ^*), then (8) implies that Q will diverge from unity (the purchasing power parity level) according to whether productivity in the tradables sector relative to the non-tradables sector is greater in the Home or in the Foreign economy.

Suppose, however, that productivity in the non-tradables sector in both the Home and Foreign economies is constant, then, taking logarithms of (8) we have:

$$q = \mu_0 + \mu_1 a_T - \mu_2 a_T^*, \tag{9}$$

where lower-case letters denote logarithms and the constant parameters μ_0, μ_1 and μ_2 are given by $\mu_0 = -(1-\gamma)a_N + (1-\gamma^*)a_N^*$, $\mu_1 = (1-\gamma) > 0$ and $\mu_2 = (1-\gamma^*) > 0$.

Equation (9) expresses the quintessence of the HBS effect: countries with relatively high levels of productivity will tend to have a less competitive equilibrium real exchange rate or, equivalently, rich countries will tend to have a higher exchange rate-adjusted price level on average.[5]

Ideally, one would like to have data on tradables sector productivity in order to investigate the HBS effect empirically. Over the long spans examined in this article, this is not available. If, however, productivity in the non-tradables sector is assumed to be

[5] Note that the HBS effect can be mitigated by having a relatively high level of productivity in the non-tradable goods sector. If however we assume $a_N \approx a_N^*$ and $\gamma \approx \gamma^*$, then $\mu_0 \approx 0$ in (9), so that variations in relative productivity in the tradable goods sectors are entirely responsible for deviations from long-run PPP. In practice, estimates of μ_0 may vary from zero simply as a reflection of the arbitrary bases used in construction of the price indices.

stagnant, then productivity in overall output will be directly proportional to tradables-sector productivity. If, in addition, we assume that the labour force is proportional to total population, then we can measure the productivity terms driving the HBS effect as the ratio of total national output – i.e. real GDP – to total population, as in the classic studies of Balassa (1964) and Officer (1976*a,b*). In our empirical analysis we maintain both of these assumptions so that $\{\mu_t\}$, the long-run equilibrium level of $\{q_t\}$ in the ESTAR model (4), is modelled as:[6]

$$\mu_t = \mu_0 + \mu_1 a_t - \mu_2 a_t^*, \tag{10}$$

where a_t^* and a_t are the logarithm of the ratio of real GDP to population in the Foreign and Home economies at time t, respectively.[7]

3. The Volatility of the Real Exchange Rate Across Nominal Regimes

As documented by Frankel and Rose (1995), there is an abundance of empirical evidence that convincingly argues that the volatility of real exchange rates tends to vary across nominal exchange rate regimes and, in particular, tends to be much higher during floating-rate regimes. Studies which have reached this conclusion from an analysis of postwar data include Mussa (1986, 1990), Eichengreen (1988), Baxter and Stockman (1989) and Flood and Rose (1995); see Taylor (1995) for a discussion of these studies. The Baxter and Stockman (1989) and Flood and Rose (1995) studies are particularly interesting in that they demonstrate that, although both real and nominal exchange rates tend to be much more volatile during floating exchange rate regimes, the underlying macro fundamental variables display no such regime-specific shifts in volatility. In a more recent and wide-ranging analysis of the exchange rates of twenty countries over a period of a hundred years, Taylor (2002) finds that the variance of the error term in simple autoregressive real exchange rate equations is almost perfectly correlated with the variance of the nominal exchange rate.

These studies suggest, therefore, that if one wishes to estimate a real exchange rate model spanning a number of nominal exchange rate regimes, it is important to allow for shifts in volatility in the error term of the empirical model since failure to do so may result, for example, in biased estimated coefficient standard errors.[8] In their long-span real exchange rate study, Lothian and Taylor (1996) explicitly acknowledge this issue and allow for shifts in volatility in a very general way by using heteroscedastic-robust estimation methods. In the present study, however, we specifically build in the possibility of shifts in volatility across nominal exchange rate regimes in designing our econometric model.

[6] In fact, as far as the productivity of the non-tradables sectors is concerned, we need only assume that there is no relative effect of non-tradables sector productivity on the real exchange rate, not necessarily that non-tradables sector productivity is constant. This follows because $\mu_0 = -(1-\gamma)a_N + (1-\gamma^*)a_N^*$ in (9). This term will be a non-zero constant if a_N and a_N^* are constant, but it will also be constant even if a_N and a_N^* are time-varying, so long as the terms $(1-\gamma)a_N$ and $(1-\gamma^*)a_N^*$ differ by a constant amount over time. This would follow where both non-tradable-sector productivity growth and the share of non-tradables in consumption were similar in the Home and Foreign economies.

[7] Although we have developed the HBS framework in terms of labour productivity rather than total factor productivity, similar results can be obtained relating to total factor productivity with a slightly more sophisticated model (Froot and Rogoff, 1995).

[8] See Lothian and McCarthy (2002) for an explict analysis of these issues.

We are particularly concerned that there may have been a downward shift in the volatility of real exchange rates during fixed nominal exchange rate regimes, such as the Bretton Woods and the interwar and classical gold standard periods. As demonstrated by Obstfeld *et al.* (2004*a,b*) and Reinhart and Rogoff (2004), however, it is important not simply to impose constraints according to official regime classifications but, rather, to use the data to determine *de facto* rather than *de jure* nominal exchange rate regimes. In particular, Obstfeld *et al.* (2004*a*) test for *de facto* adherence to the classical Gold Standard for a number of countries, on the criterion of whether or not the end-of-month exchange rate against the pound sterling stays within ±2% bands over the course of a year. On the basis of this classification, these authors find that the US dollar was *de facto* on the gold standard over the period January 1883 to June 1914, and the French franc over the period April 1872 to June 1914. Using a similar methodology, Obstfeld *et al.* (2004*b*) find that the sterling–dollar rate was fixed *de facto* for the period April 1925 to August 1931 and the sterling–franc rate for the period August 1928 to August 1931. Under the Bretton Woods System, both exchange rates were pegged against the dollar from 1946 until the breakdown of the System around 1971, although sterling was devalued in September 1949 and again in November 1967. Hence, for our annual series, the sets of years during which the sterling–dollar and franc–sterling rates were *de facto* fixed according to Obstfeld *et al.* (2004*a, b*) are given by:[9]

$$Fix(US) = \{1883 - 1913, 1926 - 1930, 1946 - 1948, 1950 - 1966, 1968 - 1970\} \quad (11)$$

$$Fix(France) = \{1872 - 1913, 1928 - 1930, 1946 - 1948, 1950 - 1966, 1968 - 1970\}. \quad (12)$$

Accordingly, if $\sigma^2_{i,t}$ is the residual variance at time t for country i ($i = US$ or $i = France$), we can allow $\sigma^2_{i,t}$ to vary across *de facto* fixed and floating nominal regimes fact by modelling it as:

$$\sigma^2_{i,t} = \sigma^2_{i,Float}[1 - I_t\{t \in Fix(i)\}] + \sigma^2_{i,Fix} I_t\{t \in Fix(i)\} \quad (13)$$

where $I_t\{.\}$ is an indicator variable, equal to unity when the statement in braces is correct. The parameters $\sigma^2_{i,Float}$ and $\sigma^2_{i,Fix}$ can then be estimated, along with those for the conditional mean, by maximum likelihood.

4. Data

Annual data on the nominal sterling–dollar and franc–sterling exchange rates, together with real GDP and population data for France, the UK and the US, were gathered for the period 1820–2001 for the UK and the US and for the period 1820–1998 for France. Details of the data sources are given in the Appendix.

The real exchange rates were constructed, in logarithmic form, as $q = s + p - p^*$ where s is the logarithm of the nominal exchange rate (dollars per pound or francs per pound), p is the logarithm of the UK price level and p^* is the logarithm of the US or French price level, as appropriate. The productivity terms were constructed as the

[9] We are grateful to Jay Shambaugh for helpful discussions and correspondence on this issue.

logarithm of the ratio of real GDP to population. All series had their sample mean removed.

5. Empirical Results

5.1. *Linear Estimation Results*

As a preliminary examination of the data, we tested for the presence of unit roots in the processes generating the real exchange rate time series, under the maintained hypothesis of linearity, using standard linear unit root tests, the results of which are reported in Table 1.[10] In each case, we are able to reject the unit root hypothesis at the 5% level or lower.

We then proceeded to estimate linear autoregressive models for each of the real exchange rates, with a lag length of one year, as suggested by examination of the partial autocorrelation function for each of the series. The results are reported in Table 2. The point estimate of the autoregressive coefficient of 0.902 for sterling–dollar is implies a half-life of adjustment of 6.78 years, while the results for franc–sterling imply a faster speed of adjustment, with a point estimate of the autoregressive coefficient of 0.831 and a corresponding half-life estimate of 3.75 years. Given that the volatility of real exchange rates implies that they must be largely driven by nominal and financial shocks which one would expect to mean revert at a much faster rate, this evidence is confirmatory of Rogoff's 'purchasing power parity puzzle' (Rogoff, 1996).

5.2. *Nonlinear Estimation Results*

Univariate estimation results. Bringing together the previous discussion on modelling nonlinearity, the Harrod-Balassa-Samuelson effect and regime-varying volatility, we can now summarise our empirical nonlinear model. We treat the UK as the Home economy and, for notational convenience, we introduce a country subscript on parameters and variables. Thus, $q_{France,t}$ is the real exchange rate between the UK and France and $q_{US,t}$ is the real exchange rate between the UK and the US. Further, treating the UK as the Home economy, Home productivity, denoted a_t in (10), becomes UK productivity at time t, denoted $a_{UK,t}$. The Foreign economy then becomes either France or the US, so that the Foreign productivity variable of (10), a_t^*, becomes either French or US productivity, denoted $a_{France,t}$ and $a_{US,t}$ respectively.

While a very general specification corresponding to (4) and (10) was initially estimated, it was found that a number of insignificant simplifying restrictions could be imposed on the model. In particular, there was no evidence of serial correlation beyond first-order on the basis of examination of the partial autocorrelation functions of the real exchange rates or from examination of the partial autocorrelation functions for the real exchange rate adjusted for relative productivity. The coefficients $\beta_{i,1}$ and $\beta_{i,1}^*$ in (4) were found to be insignificantly different from plus and minus one,

[10] Phillips and Perron (1988) and Schwert (1989) demonstrate that the Phillips-Perron non-parametric test statistics may be subject to distortion in the presence of moving-average components in the time series. Accordingly, we tested for the presence of moving-average components and could detect no statistically significant such effects in either of the real exchange rate series.

Table 1

Linear Unit Root Tests for Real Exchange Rates

(*a*) *Sterling–Dollar 1820–2001*				
τ_μ	τ_τ	Φ_1	Φ_2	Φ_3
−3.19	−3.44	4.89	4.06	6.07
$Z(\tau_\mu)$	$Z(\tau_\tau)$	$Z(\Phi_1)$	$Z(\Phi_2)$	$Z(\Phi_3)$
−3.23	−3.69	5.23	4.62	6.91
(*b*) *Sterling–Franc 1820–1998*				
τ_μ	τ_τ	Φ_1	Φ_2	Φ_3
−3.72	−3.73	6.96	4.92	7.32
$Z(\tau_\mu)$	$Z(\tau_\tau)$	$Z(\Phi_1)$	$Z(\Phi_2)$	$Z(\Phi_3)$
−3.86	−3.85	7.48	5.21	7.76

Note: Following Perron (1988), the null hypotheses for each of the test statistics are based on regressions of the form $q_t = \kappa + \lambda(t - T/2) + \delta q_{t-1} + u_t$ and $q_t = \kappa^* + \delta^* q_{t-1} + u_t^*$, where T is the sample size and u_t and u_t^* are regression residuals. The null hypotheses tested by τ_τ, Φ_2 and Φ_3 are, respectively H_A:$\delta = 1$, H_B:$(\kappa,\lambda,\delta) = (0,0,1)$ and H_C:$(\lambda,\delta) = (0,1)$. The null hypotheses tested by τ_μ and Φ_1 are, respectively, H_D:$\delta^* = 1$ and H_E:$(\kappa^*,\delta^*) = (0,1)$. In each case, $Z(.)$ denotes the corresponding non-parametric transformations of the statistic indicated in parenthesis, due to Phillips (1987) and Phillips and Perron (1988). A Newey-West window of width 4 was used for the non-parametric corrections (Newey and West, 1987), although experiments with different band-widths led to little difference in the results. The asymptotic critical values for the statistics at various test sizes are as follows (Fuller, 1976; Dickey and Fuller, 1981):

	10%	5%	2.5%	1%
$\tau_\mu, Z(\tau_\mu)$	−2.57	−2.86	−3.12	−3.43
$\tau_\tau, Z(\tau_\tau)$	−3.12	−3.41	−3.66	−3.96
$\Phi_1, Z(\Phi_1)$	3.78	4.59	5.38	6.43
$\Phi_2, Z(\Phi_2)$	4.03	4.68	5.31	6.09
$\Phi_3, Z(\Phi_3)$	5.34	6.25	7.16	8.27

respectively while the intercept term in (10) was found to be insignificantly different from zero. The delay parameter, d in (4), was chosen using the procedure suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994), as outlined in Section 2 and, as anticipated, a delay of one year appeared to capture adequately the nonlinear dynamics of the ESTAR transition function ($d = 1$). Further, the coefficient on foreign productivity, when estimated freely, was numerically close to and insignificantly different from being equal to that on domestic productivity, so that productivity was entered in relative terms. Imposing these restrictions on the model, the final parsimonious empirical specifications were therefore of the form:[11]

$$\begin{aligned}[q_{i,t} - \mu_{i,1}(a_{UK,t} - a_{i,t})] &= [q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})] \\ &\quad \times \exp\{-\theta_i[q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})]^2\} + \varepsilon_{i,t}\end{aligned} \tag{14}$$

[11] Note that the transition function in (14) is of the form exp [.] rather than the standard ESTAR transition function of the form {1 − exp [.]}, as in (4). This is because, with a first-order autoregression ($\beta_{i,j} = 0$ and $\beta^*_{i,j} = 0$, for $j > 1$), the further restrictions $\beta_{i,1} = 1$ and $\beta^*_{i,1} = -1$ imply that deviations from long-run equilibrium follow a random walk in the close neighbourhood of equilibrium, when $\exp\{-\theta_i[q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})]^2\} \approx \exp[0] = 1$, but become increasingly mean-reverting as the size of the deviation grows and $\exp\{-\theta_i[q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})]^2\} \to 0$.

Table 2

Estimated Linear Autoregressionss

(*a*) *Sterling-Dollar 1820–2001*

$$\hat{q}_{US,t} = \underset{(-1.401)}{-0.007} + \underset{(28.188)}{0.902} q_{US,t-1}$$

$$R^2 = 0.82; \quad SER = 6.45\%$$
$$AR(1) = [0.08]; \quad ARCH(1) = [0.25]; \quad HL = 6.78.$$

(*b*) *Sterling-Franc 1820–1998*

$$\hat{q}_{France,t} = \underset{(-1.286)}{-0.009} + \underset{(12.043)}{.831} q_{France,t-1}$$

$$R^2 = 0.65; \quad SER = 7.5\%;$$
$$AR(1) = [0.85]; \quad ARCH(1) = [0.00]; \quad HL = 3.75.$$

Note: Figures in parentheses below estimated coefficients are asymptotic t-ratios, calculated using heteroscedastic-consistent estimated standard errors (White, 1980); figures in square brackets are marginal significance levels. R^2 is the coefficient of determination, SER is the standard error of the regression, AR(1) is a Lagrange multiplier statistic for first-order serial correlation of the residuals, ARCH(1) is a Lagrange multiplier statistic for first-order autoregressive heteroscedasticity in the residuals, and HL is the implied estimated half-life of real exchange rate shocks.

$$\varepsilon_{i,t} \sim N(0, \sigma^2_{\mathrm{i,t}}) \tag{15}$$

$$\sigma^2_{i,t} = \sigma^2_{i,Float}[1 - I_t\{t \in Fix(i)\}] + \sigma^2_{i,Fix} I_t\{t \in Fix(i)\}. \tag{16}$$

As before, $I_t\{.\}$ is an indicator variable, equal to unity when the statement in braces is true and *Fix*(*US*) and *Fix*(*France*) are as defined in (11) and (12).

The univariate estimation results of this model, obtained by maximum likelihood, are reported in Table 3. In both cases, a good fit is indicated, with the coefficient of determination in each case improving upon that obtained using a linear model (compare Table 2). Moreover, the residual diagnostics (calculated using the residuals standardised by the square root of the estimated variance function) are in each case satisfactory. The major difference between the US and French results is that, for franc–sterling, the estimated coefficient $\hat{\mu}_{France,1}$ was found to be insignificant at the 5% level and was set to zero.

These estimation results are noteworthy for a number of reasons. First, there is significant evidence of nonlinear mean reversion, as shown by the fact that the estimated transition parameter $\hat{\theta}_i$ is in both cases strongly significantly different from zero. Note, however, that the ratio of this estimated coefficient to its standard error – the 't-ratio' – cannot be referred to the Student-t or normal distribution for purposes of inference, since under the null hypothesis $H_0 : \theta_i = 0$, $q_{i,t}$ follows a linear unit root process.[12] This introduces a singularity under the null hypothesis so that standard inference procedures cannot be used, analogously to the way in which standard inference procedures cannot be used in the usual Dickey-Fuller or augmented Dickey-

[12] In addition, under the null hypothesis, $H_0 : \theta_i = 0$, the autoregressive parameters of the nonlinear part of the specification are unidentified – see Davies (1987), Hansen (1996).

Table 3

Nonlinear Models: Single-Equation Estimates

(a) Sterling-Dollar 1820–2001

$\hat{\mu}_{US,1}$	$\hat{\theta}_{US}$	$\hat{\sigma}^2_{US,Float}$	$\hat{\sigma}^2_{US,Fix}$
0.125	2.594	0.005	0.002
(2.246)	(2.577)	(8.125)	(6.797)
	[0.009]		

$R^2 = 0.83$;
AR(1) = [0.12]; ARCH(1) = [0.46];
NL − ESTAR = [0.55]; NL − LSTAR = [0.61].

(b) Sterling-Franc 1820–1998

$\hat{\mu}_{France,1}$	$\hat{\theta}_{France}$	$\hat{\sigma}^2_{France,Float}$	$\hat{\sigma}^2_{France,Fix}$
0.00	3.064	0.007	0.003
(−)	(9.575)	(12.009)	(20.793)
	[0.001]		

$R^2 = 0.67$;
AR(1) = [0.74]; ARCH(1) = [0.83]; HBS($\mu_{France,1} = 0$) = [0.15];
NL − ESTAR = [0.67]; NL − LSTAR = [0.77].

Note: Figures in parentheses below estimated coefficients denote the ratio of the estimated coefficient to the estimated standard error (the asymptotic 't-ratio'); figures in square brackets are marginal significance levels. The marginal significance levels for the null hypotheses $H_0 : \theta_i = 0$ were calculated by Monte Carlo methods. R^2 is the coefficient of determination, SER is the standard error of the regression, AR(1) is a Lagrange multiplier statistic for first-order serial correlation of the residuals (Eitrheim and Teräsvirta, 1996) and ARCH(1) is a Lagrange multiplier statistic for first-order autoregressive heteroscedasticity in the residuals. HBS($\mu_{France,1} = 0$) is a Wald test statistic for the parameter on relative productivity to be zero in the sterling-franc equation. NL − ESTAR and NL − LSTAR are Lagrange multiplier statistics for the hypothesis of no remaining nonlinearity of the ESTAR and LSTAR (logistic smooth transition autoregressive) varieties, respectively (Eitrheim and Teräsvirta, 1996).

Fuller tests for a linear unit root. Indeed, testing the null hypothesis $H_0 : \theta_i = 0$ is tantamount to a test of the null hypothesis against the alternative hypothesis of non-linear mean reversion, rather than against the alternative of linear mean reversion. Therefore, because the distribution of the estimator of θ_i is unknown under the null hypothesis, we calculated the empirical marginal significance level of the ratio of the estimated coefficient to the estimated standard error by Monte Carlo methods under the null hypothesis that the true data generating process for the logarithm of both of the real exchange rate series was a random walk, with the parameters of the data generating process calibrated using the actual real exchange rate data over the sample period.[13] From these empirical marginal significance levels (reported in square brackets below the coefficient estimates in Table 3), we see that the estimated transition parameter is significantly different from zero with a marginal significance level of virtually zero in each case. Since these tests may be construed as nonlinear unit root tests, the results indicate strong evidence of nonlinear mean reversion for each of the real exchange rates examined over the sample period.

[13] The empirical significance levels were based on 5,000 simulations of length 280, initialised at $q_1 = 0$, from which the first 100 data points were in each case discarded. At each replication a system of ESTAR equations identical in form to those reported in Table 3 was estimated. The percentage of replications for which a 't-ratio' for the estimated transition parameters greater in absolute value than that reported in Table 3 was obtained was then taken as the empirical marginal significance level in each case.

Second, the estimated coefficient for the relative productivity term, $\hat{\mu}_{i,1}$ is strongly significantly different from zero for the case of sterling–dollar (an asymptotic *t*-ratio of nearly eight) and is correctly signed according to the Harrod-Balassa-Samuelson effect: relatively higher US productivity generates a real appreciation of the equilibrium value of the dollar against the pound. For the case of franc–sterling, however, there is no significant evidence of the HBS effect.

Joint estimation results. In order to gain efficiency in the estimation, we also estimated the US and French equations jointly by full information maximum likelihood (FIML), assuming a constant correlation coefficient between the French and US regression errors, so that the covariance matrix takes the form:

$$\begin{bmatrix} \varepsilon_{US,t} \\ \varepsilon_{France,t} \end{bmatrix} \sim \mathrm{N}(\mathrm{O}, \Sigma_{\mathrm{t}}) \tag{17}$$

$$\Sigma_t = \begin{bmatrix} \sigma^2_{US,t} & \rho \cdot \sigma_{US,t}\sigma_{France,t} \\ \rho\sigma_{US,t}\sigma_{France,t} & \sigma^2_{France,t} \end{bmatrix} \tag{18}$$

where $\sigma^2_{i,t}$ ($i = US, France$) is as defined in (15) and ρ is the constant correlation coefficient. The joint estimation results are reported in Table 4.[14]

The FIML estimates of the residual variances are almost identical to those obtained using single-equation maximum likelihood, and the estimated correlation coefficient between the US and French residual series is strongly significantly different from zero, with a point estimate of 0.169. Moreover, the HBS slope coefficient is again significantly different from zero at the 5% level only for the US, for which there is a slight increase in the point estimate of this coefficient from 0.125 to 0.140. We again calculated the empirical distribution of the 't-ratios' for the estimated transition parameters, $\hat{\theta}_i$, and they were each found to be highly significantly different from zero.[15] Perhaps the most striking aspect of the FIML estimation results, however, is the increase in the point estimates of the transition parameter, from 2.594 to 3.023 for the US and from 3.064 to 3.218 for France, implying faster speeds of mean reversion.

Calculating the average speed of mean reversion. We proceeded to gain a measure of the mean-reverting properties of the estimated nonlinear models through calculation of their implied half-lives, using the models estimated by FIML. Effectively, this involves comparing the impulse-response functions of the models with and without initial shocks. Thus, we examined the dynamic adjustment in response to shocks through impulse response functions which record the expected effect of a shock at time t on the system at time $t + j$. For a univariate linear model, the impulse response function is equivalent to a plot of the coefficients of the moving average representation; see e.g. Hamilton (1994, p. 318). Estimating the impulse response function for a

[14] Since the franc ceased to exist after 1998, the joint estimation results are for the sample period 1820–1998.

[15] The empirical distributions of the 't-ratios' for θ_i were calculated similarly to the univariate case as described above (i.e. from Monte Carlo experiments in which the data generating process is a random walk), except that they were based on joint estimation of the French and US models. Although we do not report any sophisticated residual diagnostics for the nonlinear FIML estimation results (since it is not clear what test diagnostic statistics would be applicable), for both France and the US, the fit and the fitted residuals were in fact almost identical to those of the univariate models reported in Table 3.

Table 4

Nonlinear Models: Joint Estimates

$\hat{\mu}_{US,1}$	$\hat{\theta}_{US}$	$\hat{\sigma}^2_{US,Float}$	$\hat{\sigma}^2_{US,Fix}$
0.140	3.023	0.005	0.002
(2.999)	(3.246)	(8.463)	(6.656)
	[0.009]		
	$R^2 = 0.83$		

$\hat{\mu}_{France,1}$	$\hat{\theta}_{France}$	$\hat{\sigma}^2_{France,Float}$	$\hat{\sigma}^2_{France,Fix}$
0.00	3.218	0.007	0.003
(–)	(11.121)	(12.474)	(20.283)
	[0.001]		
	$R^2 = 0.70$		

$\hat{\rho}$
0.169
(3.460)

Notes. Estimation period is 1820–1998. Estimation method is full information maximum likelihood. Figures in parentheses below estimated coefficients denote the ratio of the estimated coefficient to the estimated standard error (the asymptotic 't-ratio'); figures in square brackets are marginal significance levels. The marginal significance levels for the null hypotheses $H_0 : \theta_i = 0$ were calculated by Monte Carlo methods.

nonlinear model, however, raises special problems both of interpretation and of computation (Gallant *et al.*, 1993; Koop *et al.*, 1996). In particular, with nonlinear models, the shape of the impulse-response function is not independent with respect to either the history of the system at the time the shock occurs, the size of the shock considered, or the distribution of future exogenous innovations. Exact estimates can only be produced – for a given shock size and initial condition – by multiple integration of the nonlinear function with respect to the distribution function each of the j future innovations, which is computationally impracticable for the long forecast horizons required in impulse response analysis.

In the research reported in this article, we calculated the impulse response functions, both conditional on average initial history and conditional on initial real exchange rate equilibrium, using the Monte Carlo integration method discussed by Gallant *et al.* (1993). The basic idea is to calculate a baseline forecast for a large number of periods ahead using the estimated model. We then calculate a second forecast but this time with a shock in the initial period. The difference between the baseline forecast path and the shocked forecast path then gives the impulse response function. In each case, the forecast path is calculated by simulating the model a large number of times and taking the average. The discrete number of years it takes for the effect of the shock on the level of the real exchange rate to dissipate by 50% is then taken as the estimated half-life for that size of shock.[16]

[16] This definition of the half life may be problematic where the impulse response function is non-monotonic, since the effect of the shock on the level of the real exchange rate may drop below 50% of its initial value and then rise above it again. Fortunately, in the applications examined in this article, this was not the case.

Table 5

Estimated Half-Lives for the Nonlinear Models

(a) Conditional on average initial history						
Shock (%):	40	30	20	10	5	1
Sterling-Dollar	1	1	1	2	2	2
Sterling-Franc	1	1	1	2	2	2

(b) Conditional on initial exchange rate equilibrium						
Shock (%):	40	30	20	10	5	1
Sterling-Dollar	1	1	2	4	6	9
Sterling-Franc	1	1	2	3	4	6

Notes. Half-lives of real exchange rate shocks were calculated by Monte Carlo methods based on the model estimates reported in Table 4.

We carried out two sets of simulations. In the first, the real exchange rate is assumed to be at its long-run equilibrium prior to the shock. In the second, a large number of response functions is calculated, using every data point over the post-Bretton Woods period as the initial conditions, and these are then averaged to produce an impulse response function conditional on average initial history.[17]

The estimated half-lives of the two real exchange rate models, calculated for six sizes of shock, conditional on average initial history over the post-Bretton woods sample periods period (1973–2001 for sterling–dollar, 1973–1998 for franc–sterling), or on initial equilibrium, are shown in Table 5.[18] They illustrate well the nonlinear nature of the estimated real exchange rate models, with larger shocks mean reverting much faster than smaller shocks and shocks conditional on average history mean reverting much faster than those conditional on initial equilibrium. In particular, for shocks of 10% or less and conditional on average initial history, the half-life is in both cases two years, while larger shocks have a half-life of one year or less. These results therefore accord broadly with those reported in Taylor *et al.* (2001) and shed some light on Rogoff's (1996) 'PPP puzzle'. Only for small shocks occurring when the real exchange rate is near its equilibrium do our nonlinear models consistently yield very long half lives in the range of three to five years or more, which Rogoff (1996) terms 'glacial'. Once nonlinearity is allowed for, even small shocks of 1% to 5% have a half-life of two years or less, conditional on average history, and for larger shocks the speed of mean reversion is even faster.

6. How Important is the Harrod-Balassa-Samuelson Effect?

In Figure 1 we have plotted the sterling–dollar real exchange rate together with our measure of the Harrod-Balassa-Samuelson term, $HBS_t = \hat{\mu}_{US,1}(a_{UK,t} - a_{US,t})$, where

[17] The procedure used for conditioning on average initial history is similar to that used and described in detail in Taylor *et al.* (2001).

[18] We define a k per cent shock to the real rate as equivalent to adding log $(1 + k/100)$ to $q_{i,t}$ and we calculate the half-life as the discrete number of years taken for the impulse response function to fall below 0.5 log $(1 + k/100)$.

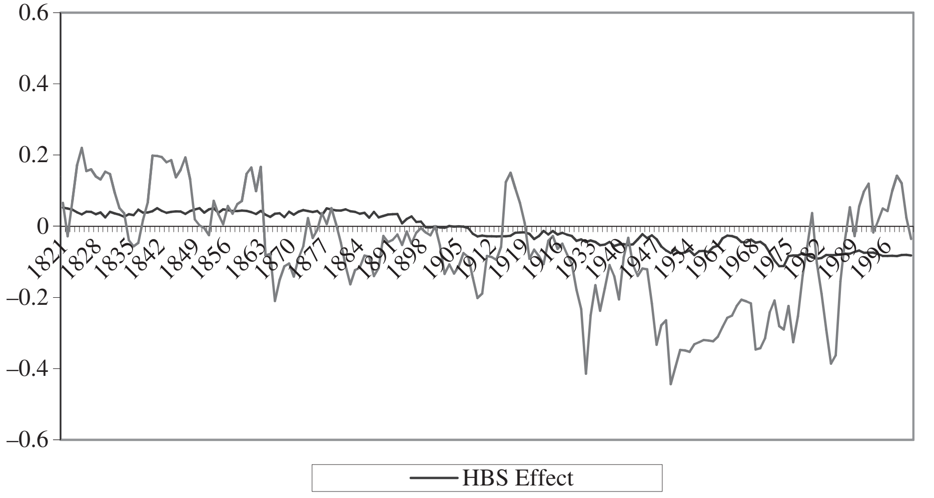

Fig. 1. *Real Sterling-Dollar and the HBS Effect*

$\hat{\mu}_{US,1}$ is the fitted value of $\mu_{US,1}$ from Table 4. It is interesting how relative productivity captures the underlying trend depreciation of the real value of sterling against the dollar over this very long period. On the other hand, this raises the question of whether this common trend is purely a statistical artefact rather than an economic relationship. Our nonlinear estimation results do indicate that the Harrod-Balassa-Samuelson effect is strongly statistically significant in explaining movements in the equilibrium real exchange rate for sterling–dollar but not for franc–sterling over the one-hundred-and-eighty-year period under investigation. However, statistical significance is not quite the same thing as economic significance. In particular, if the Harrod-Balassa-Samuelson effect has been *economically* significant, then it should be better at explaining real exchange rate movements than complex time trends and we should also perhaps expect it to account for a substantial proportion of the variation in the real exchange rate over the sample period in question. Moreover, if reversion of the real exchange rate towards its fundamental equilibrium becomes stronger over longer time horizons, then the proportion of the variation in the real exchange rate explained by deviations from that equilibrium should be an increasing function of the time horizon. We investigated each of these issues.

6.1. *Trends, Relative Productivity and the Real Exchange Rate*

In Table 6, we report the results of some simple investigations of the importance of the HBS effect for sterling–dollar. In panel (*a*) we report the results of regressing the real exchange rate onto the relative productivity term alone, $(a_{UK,t} - a_{US,t})$. The estimated slope coefficient is highly significant and the R^2 statistic reveals that the HBS effect appears to account for just over 40% of variation in the real exchange rate over the last one-hundred-and-eighty years. This accords with Rogoff's (1996) intuition that real exchange rate variation is driven largely by nominal shocks (some 60% on our measure) – although a contribution of 40% from the real side is clearly sizeable.

Table 6

The Harrod-Balassa-Samuelson Effect and the Sterling-Dollar Exchange Rate

(a) Regression of real exchange rate onto relative productivity

$$\hat{q}_t = \underset{(1.171)}{0.032} + \underset{(7.945)}{0.243}\,(a_{UK,t} - a_{US,t})$$

$$R^2 = 0.43; \quad \text{SER} = 12.85\%.$$

(b) Regression of relative productivity onto cubic trend

$$(a_{UK,t} - a_{US,t}) = \underset{(-4.890)}{-0.120} + \underset{(6.239)}{8.107 \times 10^{-3}}\,t - \underset{(-10.432)}{1.769 \times 10^{-4}}\,t^2 + \underset{(10.071)}{5.997 \times 10^{-7}}\,t^3$$

$$R^2 = 0.97; \quad \text{SER} = 9.34\%.$$

(c) Autoregression of HBS-adjusted real exchange rate with a cubic trend

$$[\hat{q}_t - \hat{\mu}_{US,1}(a_{UK,t} - a_{US,t})] = \underset{(1.002)}{0.020} + \underset{(18.279)}{0.805}\,[q_{t-1} - \hat{\mu}_{US,1}(a_{UK,t-1} - a_{US,t-1})]$$

$$+ \underset{(0.149)}{1.346 \times 10^{-4}}\,t - \underset{(-0.977)}{1.150 \times 10^{-5}}\,t^2 + \underset{(1.408)}{6.188 \times 10^{-8}}\,t^3$$

$$R^2 = 0.78; \quad \text{SER} = 6.35\%; \text{W(No Trends)} = [0.08]; \quad \text{HL} = 3.19.$$

Note: Figures in parentheses below estimated coefficients are asymptotic t-ratios, calculated using heteroscedastic-consistent estimated standard errors (White, 1980); figures in square brackets are marginal significance levels. HBS_t is the Harrod-Balassa-Samuelson effect: $HBS_t = \hat{\mu}_{US,1}(a_{UK,t} - a_{US,t})$, where $\hat{\mu}_{US,1}$ is the estimated value of $\mu_{US,1}$ in Table 4. R^2 is the coefficient of determination, SER is the standard error of the regression, W(No Trends) is a Wald test for the joint significance of the three trend parameters, and HL is the implied estimated half life of real exchange rate shocks.

In panel (*b*) of Table 5 we have reported the results of regressing relative productivity onto a cubic trend.[19] Several authors have found cubic trends to be important in explaining real exchange rate movements (Cuddington and Liang, 2000; Lothian and Taylor, 2000). In panel (*c*) of Table 5, we report the results of regressing the HBS-adjusted real exchange rate – i.e. $[q_{US,t} - \hat{\mu}_{US,1}(a_{UK,t} - a_{US,t})]$ – onto its own lagged value and the cubic trend terms. The cubic trend terms are found to be individually and jointly insignificantly different from zero, implying that the cubic trends are simply acting as an imperfect proxy for the HBS effect. In addition, note that the estimated half life of adjustment drops dramatically in the HBS-adjusted autoregression, (from the estimate of 6.78 years reported for the unadjusted sterling–dollar real exchange rate in panel (*a*) of Table 2) to 3.19 years. Although these results are clearly only indicative, especially given the importance we have demonstrated of allowing for nonlinear adjustment in real exchange rates, they are nevertheless striking.

6.2. *Explaining Real Exchange Rate Variation Due to HBS Effects at Different Time Horizons*

While the finding that HBS effects accounted for about 40% of real exchange rate variation for sterling–dollar over the whole sample period – so that some 60% of the

[19] The term 'cubic trend' is understood here to denote a function of time including terms in t and t^2 as well as t^3.

variation is due to nominal factors – it seems likely that the contribution of real factors to real exchange rate movements will vary over different time horizons. In particular, it seems reasonable to expect nominal variability to dominate mostly at shorter horizons, with real effects becoming more important at longer horizons of, say, five years or so and then diminishing in importance as the economies concerned tend to converge in real macroeconomic terms.[20] In order to investigate this possibility, we estimated long-horizon regressions of the form

$$(q_{US,t+k} - q_{US,t}) = \alpha + \gamma_k[(a_{UK,t+k} - a_{US,t+k}) - (a_{UK,t} - a_{US,t})] + v_t \quad (19)$$

where α and γ_k are regression parameters, v_t is the regression residual (which will in general be serially correlated for $k > 1$, since overlapping forecast errors will contain some common information). By regressing the change in the real exchange rate from period t to period $t + k$ onto the change in relative productivity over the same period, this regression will capture the amount of variation in the k-year change in the real exchange rate that can be explained by the k-year change in the HBS effect. Thus, if nominal rather than real effects dominate real exchange rate movements over short horizons, then we should expect a low R^2 for regressions with low values of k and increasing values of the R^2 as k increases.

The results of estimating the long-horizon regression for values of k from one to ten years are given in Table 7.[21] They are in accordance with our intuition. At the shortest

Table 7

The Short and Long-Horizon Contribution of HBS Effects to Sterling-Dollar Real Exchange Rate Variation

k	p-value of $\hat{\gamma}_k$	R_k^2
1	0.478	0.002
2	0.793	0.000
3	0.369	0.006
4	0.158	0.021
5	0.031	0.044
6	0.003	0.066
7	0.000	0.087
8	0.001	0.070
9	0.013	0.050
10	0.093	0.036

Note: The Table shows the coefficient of determination, R_k^2, and the marginal significance level (p-value) for $\hat{\gamma}_k$ in the long-horizon regression $(q_{US,t+k} - q_{US,t}) = \alpha + \gamma_k[(a_{UK,t+k} - a_{US,t+k}) - (a_{UK,t} - a_{US,t})] + v_t$ for values of k from 1 to 10. The marginal significance levels were calculated using the bootstrap algorithm described in Kilian and Taylor (2003).

[20] Indeed, this seems to be the import of Rogoff's (1996) analysis of real exchange rate movements.
[21] It is well known that asymptotic critical values for the t-test statistics for the slope coefficients in long-horizon regressions are severely biased in small samples. In order to mitigate these size distortions, empirical marginal significance levels can be calculated based on the bootstrap approximation of the finite sample distribution of the test statistic under the null hypothesis of no exchange rate predictability. The marginal significance levels reported in Table 7 were computed using the bootstrap algorithm for long-horizon regressions described in detail in Kilian and Taylor (2003).

horizon of one year, the change in productivity accounts for less than 1% of the variation in the annual change in the real exchange rate and the estimated value of γ_k has a p-value (marginal level of statistical significance) of 0.47. It is not until the time horizon reaches five years that the estimated slope parameter becomes significantly different from zero at the 5% level, with around 4% of the five-year real exchange rate change explained by the HBS effect. The significance of the HBS effect reaches its peak at seven years, when 9% of the seven-year real exchange rate change is explained, after which it declines. By the tenth year, however, relative productivity is still significant – albeit at only the 10% level – in explaining the ten-year real exchange rate change, with around 4% explained.

7. Conclusion

A reading of the empirical literature on real exchange rates and purchasing power parity suggests a number of influences worthy of investigation. The first is the effect of real variables on the equilibrium levels of real exchange rates over the long run, and in particular the influence of relative productivity differentials – the Harrod-Balassa-Samuelson effect. A second issue concerns the possibility of nonlinear adjustment of real exchange rates to their long-run equilibria. A third relates to differences in real exchange rate volatility across nominal exchange rate regimes.

We have investigated all three sets of influences in the research reported in this article. To do so, we have estimated exponential smooth transition autoregressive (ESTAR) models for real sterling–dollar and real franc–sterling exchange rates in which we include relative real *per capita* income as a proxy for relative productivity and in which we allow for possible shifts in the variance of the errors. The data set that we use spans nearly two centuries and thereby allows not only enhanced test power but also provides an environment in which the various factors that in principle can affect real exchange-rate behaviour have sufficient scope to operate.

While we find evidence of significant nonlinearities in adjustment for both exchange rates, we find significant evidence of HBS effects for sterling–dollar but not for franc–sterling. There is also evidence of shifting real exchange rate volatility for both exchange rates, with higher volatility recorded during floating nominal exchange rate regimes.

We then go on to analyse the impulse-response functions for shocks of varying magnitudes to the two real exchange rates. In both instances, these show greatly increased speeds of adjustment *vis-à-vis* those estimated with linear autoregressive models for all but the very smallest shocks. Conditional on average initial history, the estimated half-lives for large shocks of 20% or more are only one year; for small shocks in the range of 1% to 5% they range from one to two years depending upon the exact magnitude of the shocks.

While the HBS effect is able to explain some 40% of the variation in the level of the sterling–dollar real exchange rate over the whole sample period, we found that the influence of real effects on the real exchange rate varies according to the time horizon considered. In particular, long-horizon regressions of the *k*-year change in the real exchange rate onto the *k*-year change in relative productivity revealed that at the shortest horizon of one year, HBS effects account for only a tiny proportion of the change in the real exchange rate. The proportion explained increases with the length of the time horizon, however, until it peaks at the seven-year horizon, when HBS effects explain around 9% of the seven-year change in the real exchange rate.

Finally, we should end with a note of caution. Although we have found statistically significant evidence of the HBS effect for the UK-US real exchange rate, we failed to find any significant evidence of the HBS effect for the UK-French real exchange rate. One reason why this may be the case is because of parallels in industrial development between the UK and France. In particular, although the industrial revolution began in the UK, French industrialisation did not lag behind that of the UK so far as did US industrialisation for much of the nineteenth century, nor did French productivity overtake UK productivity as markedly as did US productivity in the twentieth. Nevertheless, this is an issue that warrants further research.

This research might also be fruitfully extended in a number of other directions. First, investigation of the Harrod-Balassa-Samuelson effect in a nonlinear framework could be carried out for other countries, especially those that have experienced high rates of growth relative to the base country. Second, the analysis could be repeated, focusing on the recent floating-rate period, and perhaps employing nonlinear panel estimation methods for a group of countries. Third, the framework used in this article could be extended to a multivariate nonlinear system involving nominal exchange rates and relative prices as well as productivity differentials, in order to examine the relative speed of adjustment of nominal exchange rates and relative prices to deviations from the equilibrium real exchange rate.

Appendix: Data and Sources

France

Nominal exchange rate and price index. Data for the period 1820–1992 were taken from Lothian and Taylor (1996) and updated from the International Monetary Fund's *International Financial Statistics (IFS)* database. For a full description of the earlier data for all three countries and their sources, see the appendix to Lothian and Taylor (1996).

Real income. GDP data for 1820–1870 came from Toutain (1997) and were spliced with GDP data from Maddison (1995) for 1870–1994 and with data from IFS thereafter. (The Toutain and Maddison data are also available at the website of Jean Bourdon, Institut de Recherche sur l'Education Sociologie et Economie de l'Education at the University of Bourgogne: http://perso.orange.fr/jbourdon/CsectionB.htm.)

Population. Mitchell (1998) for 1820–1869, from Maddison (1995) for 1870–1994 and from IFS thereafter.

United Kingdom

Price index. Data for the period 1820–1992 were taken from Lothian and Taylor (1996) and updated from the IFS.

Real income. Data for GDP the period prior to 1864 were derived by linear interpolation from the decadal estimates in Clark (2001) and were spliced with data for GDP at factor cost from Feinstein (1972) for 1864–69, with GDP data from Maddison (1995) for 1870–1994 and from the IFS for the period thereafter.

Population. Data for 1820–1980 came from Mitchell (1988) and from the IFS thereafter.

United States

Nominal exchange rate and price index. Data for the period 1820–1992 were taken from Lothian and Taylor (1996) and from IFS thereafter.

Real income. Data for GDP came from Officer (2002) for 1820–1869 and were linked to GDP data from Maddison (1995) for 1870–1994 and from the IFS thereafter.

Population. Populstat (http://www.library.uu.nl/wesp/populstat/populhome.html) for 1820–1994 and from the IFS thereafter.

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